# MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 3 **MATHEMATICS 1001** WINTER 2023

## SOLUTIONS

[5] 1. (a) First we rewrite the denominator by completing the square:

$$\begin{aligned} 16x^2 - 24x + 34 &= 16\left[x^2 - \frac{3}{2}x + \frac{17}{8}\right] \\ &= 16\left[\left(x^2 - \frac{3}{2}x + \frac{9}{16}\right) + \frac{17}{8} - \frac{9}{16}\right] \\ &= 16\left[\left(x - \frac{3}{4}\right)^2 + \frac{25}{16}\right] \\ &= 16\left(x - \frac{3}{4}\right)^2 + 25 \\ &= (4x-3)^2 + 25. \end{aligned}$$

Thus

$$\int \frac{1}{16x^2 - 24x + 34}\,dx = \frac{1}{(4x-3)^2 + 25}\,dx.$$

Now let $u = 4x - 3$ so $du = 4\,dx$ and $\frac{1}{4}\,du = dx$. The integral becomes

$$\begin{aligned} \int \frac{1}{16x^2 - 24x + 34}\,dx &= \frac{1}{4}\int \frac{1}{u^2 + 25}\,du \\ &= \frac{1}{4}\cdot\frac{1}{5}\arctan\left(\frac{u}{5}\right) + C \\ &= \frac{1}{20}\arctan\left(\frac{4x-3}{5}\right) + C. \end{aligned}$$

[3] (b) Observe that

$$\int \frac{e^t}{\sqrt{7 - e^{2t}}}\,dt = \int \frac{e^t}{\sqrt{7 - (e^t)^2}}\,dt.$$

So let $u = e^t$ and $du = e^t\,dt$. The integral becomes

$$\begin{aligned} \int \frac{e^t}{\sqrt{7 - e^{2t}}}\,dt &= \int \frac{1}{\sqrt{7 - u^2}}\,du \\ &= \arcsin\left(\frac{u}{\sqrt{7}}\right) + C \\ &= \arcsin\left(\frac{e^t}{\sqrt{7}}\right) + C \\ &= \arcsin\left(\frac{\sqrt{7}e^t}{7}\right) + C. \end{aligned}$$

[3] 2. (a) Let $w = \ln^2(x)$ so $dw = \dfrac{2\ln(x)}{x}\,dx$. Let $dv = dx$ so $v = x$. Then

$$\int \ln^2(x)\,dx = x\ln^2(x) - \int x \cdot \frac{2\ln(x)}{x}\,dx$$
$$= x\ln^2(x) - 2\int \ln(x)\,dx.$$

Here we can use integration by parts again (or our result from class) to find that

$$\int \ln(x)\,dx = x\ln(x) - x + C,$$

and so

$$\int \ln^2(x)\,dx = x\ln^2(x) - 2[x\ln(x) - x] + C$$
$$= x\ln^2(x) - 2x\ln(x) + 2x + C.$$

[4] (b) Let $w = \arctan(x)$ so $dw = \dfrac{1}{1+x^2}\,dx$. Let $dv = x\,dx$ so $v = \frac{1}{2}x^2$. Then

$$\int x\arctan(x)\,dx = \frac{1}{2}x^2\arctan(x) - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx.$$

To evaluate the remaining integral, in principal we need long division (because this is an improper rational function). However, because the numerator and denominator are so similar, we can more straightforwardly write:

$$\int \frac{x^2}{1+x^2}\,dx = \int \frac{1+x^2-1}{1+x^2}\,dx$$
$$= \int \left(\frac{1+x^2}{1+x^2} - \frac{1}{1+x^2}\right)dx$$
$$= \int \left(1 - \frac{1}{1+x^2}\right)dx$$
$$= x - \arctan(x) + C.$$

Either way, we now have

$$\int x\arctan(x)\,dx = \frac{1}{2}x^2\arctan(x) - \frac{1}{2}[x - \arctan(x)] + C$$
$$= \frac{1}{2}x^2\arctan(x) - \frac{1}{2}x + \frac{1}{2}\arctan(x) + C.$$

[5] (c) Let $w = \sinh(3t)$ so $dw = 3\cosh(3t)\,dt$. Let $dv = \cos(7t)\,dt$ so $v = \frac{1}{7}\sin(7t)$. Then

$$\int \sinh(3t)\cos(7t)\,dt = \frac{1}{7}\sinh(3t)\sin(7t) - \frac{3}{7}\int \cosh(3t)\sin(7t)\,dt.$$

Now we use integration by parts again. Let $w = \cosh(3t)$ so $dw = 3\sinh(3t)$. Let $dv = \sin(7t)\,dt$ so $v = -\frac{1}{7}\cos(7t)$. We have

$$\int \cosh(3t)\sin(7t)\,dt = -\frac{1}{7}\cosh(3t)\cos(7t) + \frac{3}{7}\int \sinh(3t)\cos(7t)\,dt.$$

Now we note that the original integral has reoccurred, so putting all of this together we have

$$\begin{aligned}
\int \sinh(3t)\cos(7t)\,dt &= \frac{1}{7}\sinh(3t)\sin(7t) \\
&\qquad - \frac{3}{7}\left[-\frac{1}{7}\cosh(3t)\cos(7t) + \frac{3}{7}\int \sinh(3t)\cos(7t)\,dt\right] \\
&= \frac{1}{7}\sinh(3t)\sin(7t) + \frac{3}{49}\cosh(3t)\cos(7t) - \frac{9}{49}\int \sinh(3t)\cos(7t)\,dt \\
\frac{58}{49}\int \sinh(3t)\cos(7t)\,dt &= \frac{1}{7}\sinh(3t)\sin(7t) + \frac{3}{49}\cosh(3t)\cos(7t) + C \\
\int \sinh(3t)\cos(7t)\,dt &= \frac{7}{58}\sinh(3t)\sin(7t) + \frac{3}{58}\cosh(3t)\cos(7t) + C.
\end{aligned}$$