MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 1 | **MATHEMATICS 1001** | WINTER 2023

SOLUTIONS

[4] 1. (a) We can expand the numerator and then simplify to obtain

$$\begin{aligned}\int \frac{(2x-1)(x^2+3)}{x^2}\,dx &= \int \frac{2x^3-x^2+6x-3}{x^2}\,dx \\ &= 2\int x\,dx - \int dx + 6\int \frac{1}{x}\,dx - 3\int x^{-2}\,dx \\ &= 2\left[\frac{x^2}{2}\right] - x + 6\ln|x| - 3\left[\frac{x^{-1}}{-1}\right] + C \\ &= x^2 - x + 6\ln|x| + \frac{3}{x} + C.\end{aligned}$$

[3] (b) Using the properties of logarithms, we can write

$$\begin{aligned}\int \frac{\ln(x^5)}{\ln\left(\sqrt{x}\right)}\,dx &= \int \frac{5\ln(x)}{\frac{1}{2}\ln(x)}\,dx \\ &= 10\int dx \\ &= 10x + C.\end{aligned}$$

[3] (c) First observe that

$$\sin(u)\cot(u)\sec(u) = \sin(u)\cdot\frac{\cos(u)}{\sin(u)}\cdot\frac{1}{\cos(u)} = 1.$$

Thus we simply have

$$\int \sin(u)\cot(u)\sec(u)\,du = \int du = u + C.$$

[3] (d) Since

$$\int x^{20}\,dx = \frac{x^{21}}{21} + C,$$

we can write

$$\begin{aligned}\int \left(\frac{x}{7}+8\right)^{20} dx &= \frac{\left(\frac{x}{7}+8\right)^{21}}{\frac{1}{7}\cdot 21} + C \\ &= \frac{1}{3}\left(\frac{x}{7}+8\right)^{21} + C.\end{aligned}$$

[3] (e) Since

$$\int x^{-2}\,dx = \frac{x^{-1}}{-1} + C,$$

we obtain

$$\begin{aligned}
\int \frac{1}{(7x+8)^2}\,dx &= \int (7x+8)^{-2}\,dx \\
&= \frac{(7x+8)^{-1}}{7\cdot(-1)} + C \\
&= -\frac{1}{7(7x+8)} + C \\
&= -\frac{1}{49x+56} + C.
\end{aligned}$$

[4] (f) Since

$$\int \sec(x)\tan(x) = \sec(x) + C,$$

we have

$$\begin{aligned}
\int \sec\left(\frac{3x+4}{6}\right)\tan\left(\frac{3x+4}{6}\right)\,dx &= \int \sec\left(\frac{1}{2}x+\frac{2}{3}\right)\tan\left(\frac{1}{2}x+\frac{2}{3}\right)\,dx \\
&= \frac{\sec\left(\frac{1}{2}x+\frac{2}{3}\right)}{\frac{1}{2}} + C \\
&= 2\sec\left(\frac{1}{2}x+\frac{2}{3}\right) + C.
\end{aligned}$$