

MEMORIAL UNIVERSITY OF NEWFOUNDLAND  
DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 1.3

Math 1001 Worksheet

WINTER 2023

SOLUTIONS

1. (a) Note that

$$\int \frac{1}{\sqrt{9-36x^2}} dx = \int \frac{1}{\sqrt{3^2-(6x)^2}} dx$$

so let  $u = 6x$  and  $\frac{1}{6} du = dx$ . The integral becomes

$$\begin{aligned} \int \frac{1}{\sqrt{9-36x^2}} dx &= \frac{1}{6} \int \frac{1}{\sqrt{3^2-u^2}} du \\ &= \frac{1}{6} \left[ \arcsin\left(\frac{u}{3}\right) \right] + C \\ &= \frac{1}{6} \arcsin(2x) + C. \end{aligned}$$

(b) We have

$$\int \frac{1}{-9x^2-2} dx = - \int \frac{1}{(3x)^2+2} dx$$

so let  $u = 3x$  and  $\frac{1}{3} du = dx$ . Then

$$\begin{aligned} \frac{1}{-9x^2-2} dx &= -\frac{1}{3} \int \frac{1}{u^2+(\sqrt{2})^2} du \\ &= -\frac{1}{3} \left[ \frac{1}{\sqrt{2}} \arctan\left(\frac{u}{\sqrt{2}}\right) \right] + C \\ &= -\frac{\sqrt{2}}{6} \arctan\left(\frac{3\sqrt{2}x}{2}\right) + C. \end{aligned}$$

(c) We first complete the square, giving

$$x^2 + 4x = (x^2 + 4x + 4) - 4 = (x + 2)^2 - 2^2$$

so we can write

$$\int \frac{1}{(x+2)\sqrt{x^2+4x}} dx = \int \frac{1}{(x+2)\sqrt{(x+2)^2-2^2}} dx.$$

Let  $u = x + 2$  so  $du = dx$ . Now we have

$$\begin{aligned} \int \frac{1}{(x+2)\sqrt{x^2+4x}} dx &= \int \frac{1}{u\sqrt{u^2-4}} du \\ &= \frac{1}{2} \operatorname{arcsec}\left(\frac{u}{2}\right) + C \\ &= \frac{1}{2} \operatorname{arcsec}\left(\frac{x+2}{2}\right) + C. \end{aligned}$$

(d) Let  $u = \cos(\theta)$  so  $-du = \sin(\theta) d\theta$ . The integral becomes

$$\begin{aligned}\int \frac{\sin(\theta)}{\cos^2(\theta) + 4} d\theta &= -\int \frac{1}{u^2 + 2^2} du \\ &= -\frac{1}{2} \arctan\left(\frac{u}{2}\right) + C \\ &= -\frac{1}{2} \arctan\left(\frac{\cos(\theta)}{2}\right) + C.\end{aligned}$$

(e) Note that we can write

$$\int \frac{1}{x\sqrt{81x^2 - 49}} dx = \int \frac{1}{x\sqrt{(9x)^2 - 7^2}} dx.$$

Let  $u = 9x$  so  $\frac{1}{9} du = dx$  and  $x = \frac{1}{9}u$ . The integral becomes

$$\begin{aligned}\frac{1}{9} \int \frac{1}{\left(\frac{1}{9}u\right)\sqrt{u^2 - 7^2}} du &= \int \frac{1}{u\sqrt{u^2 - 7^2}} du \\ &= \frac{1}{7} \operatorname{arcsec}\left(\frac{u}{7}\right) + C \\ &= \frac{1}{7} \operatorname{arcsec}\left(\frac{9x}{7}\right) + C.\end{aligned}$$

(f) Completing the square gives

$$\begin{aligned}\frac{1}{9}x^2 - 4x + 40 &= \frac{1}{9}[x^2 - 36x + 360] \\ &= \frac{1}{9}[(x^2 - 36x + 324) + 36] \\ &= \frac{1}{9}[(x - 18)^2 + 36] \\ &= \left(\frac{x}{3} - 6\right)^2 + 2^2.\end{aligned}$$

The integral can now be written

$$\int \frac{1}{\frac{1}{9}x^2 - 4x + 40} dx = \int \frac{1}{\left(\frac{x}{3} - 6\right)^2 + 2^2} dx.$$

Let  $u = \frac{x}{3} - 6$  so  $3 du = dx$ . Now we have

$$\begin{aligned}\int \frac{1}{\frac{1}{9}x^2 - 4x + 40} dx &= 3 \int \frac{1}{u^2 + 2^2} du \\ &= 3 \left[ \frac{1}{2} \arctan\left(\frac{u}{2}\right) + C \right] \\ &= \frac{3}{2} \arctan\left(\frac{\frac{x}{3} - 6}{2}\right) + C \\ &= \frac{3}{2} \arctan\left(\frac{x - 18}{6}\right) + C.\end{aligned}$$

(g) Let  $u = e^{2x}$  so  $\frac{1}{2} du = e^{2x} dx$ . The integral can be written

$$\int \frac{e^{2x}}{\sqrt{-e^{4x} - 10e^{2x} - 24}} dx = \frac{1}{2} \int \frac{du}{\sqrt{-u^2 - 10u - 24}}.$$

But note that

$$\begin{aligned}-u^2 - 10u - 24 &= -(u^2 + 10u + 24) = -[(u^2 + 10u + 25) - 1] \\ &= -[(u + 5)^2 - 1] = 1 - (u + 5)^2\end{aligned}$$

so the integral becomes

$$\int \frac{e^{2x}}{\sqrt{-e^{4x} - 10e^{2x} - 24}} dx = \frac{1}{2} \int \frac{du}{\sqrt{1 - (u + 5)^2}}.$$

Next let  $v = u + 5$ , so  $dv = du$ . Now we obtain

$$\begin{aligned}\int \frac{e^{2x}}{\sqrt{-e^{4x} - 10e^{2x} - 24}} dx &= \frac{1}{2} \int \frac{1}{\sqrt{1 - v^2}} dv \\ &= \frac{1}{2} [\arcsin(v)] + C \\ &= \frac{1}{2} \arcsin(u + 5) + C \\ &= \frac{1}{2} \arcsin(e^{2x} + 5) + C.\end{aligned}$$

(h) First we rewrite the integral as

$$\int \frac{9x - 6}{\sqrt{1 - 25x^2}} dx = 9 \int \frac{x}{\sqrt{1 - 25x^2}} dx - 6 \int \frac{dx}{\sqrt{1^2 - (5x)^2}}.$$

For the first integral, let  $u = 1 - 25x^2$  so  $-\frac{1}{50} du = x dx$ . For the second integral, let  $v = 5x$  so  $\frac{1}{5} dv = dx$ . Then the integral becomes

$$\begin{aligned} \int \frac{9x - 6}{\sqrt{1 - 25x^2}} dx &= -\frac{9}{50} \int u^{-\frac{1}{2}} du - \frac{6}{5} \int \frac{dv}{\sqrt{1^2 - v^2}} \\ &= -\frac{9}{50} [2\sqrt{u}] - \frac{6}{5} \arcsin(v) + C \\ &= -\frac{9}{25} \sqrt{1 - 25x^2} - \frac{6}{5} \arcsin(5x) + C. \end{aligned}$$

(i) Let  $u = x^2$  so

$$\begin{aligned} 4x^4 + 4x^2 + 17 &= 4u^2 + 4u + 17 \\ &= 4 \left[ u^2 + u + \frac{17}{4} \right] \\ &= 4 \left[ \left( u^2 + u + \frac{1}{4} \right) + 4 \right] \\ &= 4 \left[ \left( u + \frac{1}{2} \right)^2 + 4 \right] \\ &= (2u + 1)^2 + 4^2 \\ &= (2x^2 + 1)^2 + 4^2. \end{aligned}$$

Now the integral can be written

$$\int \frac{2x}{4x^4 + 4x^2 + 17} dx = \int \frac{2x}{(2x^2 + 1)^2 + 4^2} dx.$$

Then set  $v = 2x^2 + 1$  so  $\frac{1}{2} dv = 2x dx$ . We obtain

$$\begin{aligned} \int \frac{2x}{4x^4 + 4x^2 + 17} dx &= \frac{1}{2} \int \frac{dv}{v^2 + 4^2} \\ &= \frac{1}{2} \left[ \frac{1}{4} \arctan \left( \frac{v}{4} \right) \right] + C \\ &= \frac{1}{8} \arctan \left( \frac{2x^2 + 1}{4} \right) + C. \end{aligned}$$