

$$\text{eg } \textcircled{1} \int (x^2-4) \sqrt{\frac{1}{2}x+1} dx$$

$$\text{Let } u = \frac{1}{2}x+1$$

$$du = \frac{1}{2} dx \rightarrow 2du = dx$$

$$\text{Then } \frac{1}{2}x = u-1$$

$$x = 2u-2$$

$$x^2 = (2u-2)^2 = 4u^2 - 8u + 4$$

$$x^2-4 = 4u^2 - 8u$$

The integral becomes

$$\int (x^2-4) \sqrt{\frac{1}{2}x+1} dx = \int (4u^2-8u) \sqrt{u} \cdot 2du$$

$$= 8 \int (u^2-2u) \sqrt{u} du$$

$$= 8 \int (u^{5/2} - 2u^{3/2}) du$$

$$= 8 \left[ \frac{u^{7/2}}{7/2} - 2 \cdot \frac{u^{5/2}}{5/2} \right] + C$$

$$\boxed{= \frac{16}{7} \left(\frac{1}{2}x+1\right)^{7/2} - \frac{32}{5} \left(\frac{1}{2}x+1\right)^{5/2} + C}$$

$$\textcircled{2} \int \sin(2x) \sin(x) dx$$

$$= \int 2 \sin(x) \cos(x) \cdot \sin(x) dx$$

$$= 2 \int \sin^2(x) \cos(x) dx$$

$$= 2 \int u^2 du$$

$$= 2 \left[ \frac{u^3}{3} \right] + C$$

$$\boxed{= \frac{2}{3} \sin^3(x) + C}$$

$$\text{Let } u = \sin(x) \\ du = \cos(x) dx$$