MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 4 **MATHEMATICS 1001** FALL 2025

SOLUTIONS

[5] 1. (a) We form a regular partition where

$$\Delta x = \frac{3-2}{n} = \frac{1}{n}$$

and choose the sample point

$$x_i^* = x_i = 2 + \frac{i}{n}.$$

Thus

$$\begin{aligned} f(x_i^*) &= f\left(2 + \frac{i}{n}\right) \\ &= 5 + 8\left(2 + \frac{i}{n}\right) - 3\left(2 + \frac{i}{n}\right)^2 \\ &= 9 - \frac{4i}{n} - \frac{3i^2}{n^2}. \end{aligned}$$

Now we have

$$\begin{aligned} A &= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x \\ &= \lim_{n\to\infty} \sum_{i=1}^{n} \left[9 - \frac{4i}{n} - \frac{3i^2}{n^2}\right] \cdot \frac{1}{n} \\ &= \lim_{n\to\infty} \left[\frac{9}{n}\sum_{i=1}^{n} 1 - \frac{4}{n^2}\sum_{i=1}^{n} i - \frac{3}{n^3}\sum_{i=1}^{n} i^2\right] \\ &= \lim_{n\to\infty} \left[\frac{9}{n}\cdot n - \frac{4}{n^2}\cdot\frac{n(n+1)}{2} - \frac{3}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right] \\ &= 9 - 2 - 1 \\ &= 6. \end{aligned}$$

[5] (b) We form a regular partition where

$$\Delta x = \frac{2-(-2)}{n} = \frac{4}{n}$$

and choose the sample point

$$x_i^* = x_i = -2 + \frac{4i}{n}.$$

Thus

$$\begin{aligned} f(x_i^*) &= f\left(-2+\frac{4i}{n}\right) \\ &= \left(-2+\frac{4i}{n}\right)^2\left(-2+\frac{4i}{n}+2\right) \\ &= \frac{16i}{n}-\frac{64i^2}{n^2}+\frac{64i^3}{n^3}. \end{aligned}$$

Now we have

$$\begin{aligned} A &= \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x \\ &= \lim_{n\to\infty}\sum_{i=1}^{n}\left[\frac{16i}{n}-\frac{64i^2}{n^2}+\frac{64i^3}{n^3}\right]\cdot\frac{4}{n} \\ &= \lim_{n\to\infty}\left[\frac{64}{n^2}\sum_{i=1}^{n} i-\frac{256}{n^3}\sum_{i=1}^{n} i^2+\frac{256}{n^4}\sum_{i=1}^{n} i^3\right] \\ &= \lim_{n\to\infty}\left[\frac{64}{n^2}\cdot\frac{n(n+1)}{2}-\frac{256}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}+\frac{256}{n^4}\cdot\frac{n^2(n+1)^2}{4}\right] \\ &= 32-\frac{256}{3}+64 \\ &= \frac{32}{3}. \end{aligned}$$

[5] 2. For the indicated sample point, the width of the ith partition is given by

$$\Delta x_i = x_i - x_{i-1} = \frac{4i^2}{n^2}-\frac{4(i-1)^2}{n^2} = \frac{8i-4}{n^2}.$$

Since

$$f(x_i^*) = f\left(\frac{4i^2}{n^2}\right) = \frac{2i}{n},$$

we have

$$\begin{aligned} A &= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x_i \\ &= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{2i}{n} \cdot \frac{8i-4}{n^2} \\ &= \lim_{n\to\infty} \left[\frac{16}{n^3} \sum_{i=1}^{n} i^2 - \frac{8}{n^3} \sum_{i=1}^{n} i \right] \\ &= \lim_{n\to\infty} \left[\frac{16}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} - \frac{8}{n^3} \cdot \frac{n(n+1)}{2} \right] \\ &= \frac{16}{3} - 0 \\ &= \frac{16}{3}. \end{aligned}$$

[5] 3. We form a regular partition of $[0, 2]$ into n subintervals of length

$$\Delta x = \frac{2-0}{n} = \frac{2}{n}.$$

As the sample point, we choose

$$x_i^* = x_i = 0 + i \cdot \frac{2}{n} = \frac{2i}{n}$$

so

$$f(x_i^*) = f\left(\frac{2i}{n}\right) = \frac{8i}{n} - \frac{8i^3}{n^3}.$$

Thus

$$\begin{aligned} \int_0^2 (4x - x^2)\, dx &= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x \\ &= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{8i}{n} - \frac{8i^3}{n^3} \right] \cdot \frac{2}{n} \\ &= \lim_{n\to\infty} \left[\frac{16}{n^2} \sum_{i=1}^{n} i - \frac{16}{n^4} \sum_{i=1}^{n} i^3 \right] \\ &= \lim_{n\to\infty} \left[\frac{16}{n^2} \cdot \frac{n(n+1)}{2} - \frac{16}{n^4} \cdot \frac{n^2(n+1)^2}{4} \right] \\ &= 8 - 4 \\ &= 4. \end{aligned}$$