

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 3.1

Math 1001 Worksheet

FALL 2025

SOLUTIONS

1. (a) The partial fraction decomposition is

$$\frac{3x-2}{x^2-x} = \frac{3x-2}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1} \implies 3x-2 = A(x-1) + Bx.$$

When $x = 0$, $-A = -2$ so $A = 2$. When $x = 1$, we get $B = 1$. Thus the integral becomes

$$\begin{aligned} \int \frac{3x-2}{x^2-x} dx &= \int \left[\frac{2}{x} + \frac{1}{x-1} \right] dx \\ &= 2 \ln|x| + \ln|x-1| + C. \end{aligned}$$

- (b) The partial fraction decomposition is

$$\frac{2-x}{x^2+7x+10} = \frac{2-x}{(x+5)(x+2)} = \frac{A}{x+5} + \frac{B}{x+2} \implies 2-x = A(x+2) + B(x+5).$$

When $x = -5$, $-3A = 7$ so $A = -\frac{7}{3}$. When $x = -2$, $3B = 4$ so $B = \frac{4}{3}$. The integral becomes

$$\begin{aligned} \int \frac{2-x}{x^2+7x+10} dx &= \int \left[\frac{-\frac{7}{3}}{x+5} + \frac{\frac{4}{3}}{x+2} \right] dx \\ &= -\frac{7}{3} \ln|x+5| + \frac{4}{3} \ln|x+2| + C. \end{aligned}$$

- (c) The partial fraction decomposition is

$$\begin{aligned} \frac{6x^3+x^2+25x+4}{x^4+7x^2+12} &= \frac{6x^3+x^2+25x+4}{(x^2+4)(x^2+3)} = \frac{Ax+B}{x^2+4} + \frac{Cx+D}{x^2+3} \\ 6x^3+x^2+25x+4 &= (Ax+B)(x^2+3) + (Cx+D)(x^2+4). \end{aligned}$$

Unfortunately, there is no simple way to solve for the constants A , B , C and D , so we'll set up a system of four equations in the four unknowns. When $x = 0$, we have

$$3B + 4D = 4.$$

When $x = 1$, we have

$$4A + 4B + 5C + 5D = 36.$$

When $x = -1$, we have

$$4B - 4A + 5D - 5C = -26.$$

When $x = 2$, we have

$$14A + 7B + 16C + 8D = 106.$$

Adding the second and third equations gives

$$8B + 10D = 10 \implies 4B + 5D = 5.$$

Since the first equation implies that

$$D = 1 - \frac{3}{4}B,$$

we have

$$4B + 5\left(1 - \frac{3}{4}B\right) = 5 \implies \frac{1}{4}B = 0$$

so $B = 0$ and hence $D = 1$. Now the last two equations become

$$-4A - 5C = -31 \implies A = \frac{31}{4} - \frac{5}{4}C$$

and

$$14A + 16C = 98 \implies 7\left(\frac{31}{4} - \frac{5}{4}C\right) + 8C = 49 \implies -\frac{3}{4}C = -\frac{21}{4}$$

so $C = 7$ and therefore $A = -1$. The integral becomes

$$\begin{aligned} \int \frac{6x^3 + x^2 + 25x + 4}{x^4 + 7x^2 + 12} dx &= \int \left[\frac{-x}{x^2 + 4} + \frac{7x + 1}{x^2 + 3} \right] dx \\ &= \int \left[-\frac{x}{x^2 + 4} + \frac{7x}{x^2 + 3} + \frac{1}{x^2 + 3} \right] dx \\ &= -\frac{1}{2} \ln(x^2 + 4) + \frac{7}{2} \ln(x^2 + 3) + \frac{\sqrt{3}}{3} \arctan\left(\frac{\sqrt{3}x}{3}\right) + C. \end{aligned}$$

Here, the first two integrals can be evaluated by u -substitution with $u = x^2 + 4$ and $u = x^2 + 3$, respectively.

(d) The partial fraction decomposition is

$$\begin{aligned} \frac{-2x^3 + 12x^2 + 162x}{x^4 - 81} &= \frac{-2x^3 + 12x^2 + 162x}{(x + 3)(x - 3)(x^2 + 9)} = \frac{A}{x - 3} + \frac{B}{x + 3} + \frac{Cx + D}{x^2 + 9} \\ -2x^3 + 12x^2 + 162x &= A(x + 3)(x^2 + 9) + B(x - 3)(x^2 + 9) \\ &\quad + (Cx + D)(x + 3)(x - 3). \end{aligned}$$

When $x = 3$, $108A = 540$ so $A = 5$. When $x = -3$, $-108B = -324$ so $B = 3$. When $x = 0$, $27A - 27B - 9D = 54 - 9D = 0$ so $D = 6$. And when $x = 1$, $40A - 20B - 8C - 8D = 92 - 8C = 172$ so $C = -10$. The integral becomes

$$\begin{aligned} \int \frac{-2x^3 + 12x^2 + 162x}{x^4 - 81} dx &= \int \left[\frac{5}{x-3} + \frac{3}{x+3} + \frac{-10x+6}{x^2+9} \right] dx \\ &= \int \left[\frac{5}{x-3} + \frac{3}{x+3} - \frac{10x}{x^2+9} + \frac{6}{x^2+9} \right] dx \\ &= 5 \ln|x-3| + 3 \ln|x+3| - 5 \ln(x^2+9) + 2 \arctan\left(\frac{x}{3}\right) + C. \end{aligned}$$

Here, the third integral can be evaluated by u -substitution with $u = x^2 + 9$.

2. (a) We have

$$\begin{aligned} \frac{7+5x-2x^2}{(x-3)^3} &= \frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{C}{(x-3)^3} \\ 7+5x-2x^2 &= A(x-3)^2 + B(x-3) + C \\ &= Ax^2 - 6Ax + 9A + Bx - 3B + C \\ &= (9A - 3B + C) + (B - 6A)x + Ax^2. \end{aligned}$$

(b) Comparing the coefficients of x^2 , we immediately have $A = -2$. Comparing the coefficients of x , we have $B - 6A = B + 12 = 5$ so $B = -7$. Finally, comparing the constant coefficients, we have $9A - 3B + C = 3 + C = 7$ so $C = 4$.

(c) By the method of partial fractions, we can write

$$\begin{aligned} \int \frac{7+5x-2x^2}{(x-3)^3} dx &= \int \left[\frac{-2}{x-3} + \frac{-7}{(x-3)^2} + \frac{4}{(x-3)^3} \right] dx \\ &= -2 \ln|x-3| + \frac{7}{x-3} - \frac{2}{(x-3)^2} + C. \end{aligned}$$