MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 6 | **MATHEMATICS 1000** | FALL 2025

SOLUTIONS

[5] 1. Using implicit differentiation with respect to t,

$$\begin{aligned}
\frac{d}{dt}[t^2w^3 + 9t] &= \frac{d}{dt}[\sin(tw) - 4] \\
2tw^3 + 3w^2\frac{dw}{dt}\cdot t^2 + 9 &= \cos(tw)\left[1\cdot w + \frac{dw}{dt}\cdot t\right] - 0 \\
2tw^3 + 3t^2w^2\frac{dw}{dt} + 9 &= w\cos(tw) + t\cos(tw)\frac{dw}{dt} \\
3t^2w^2\frac{dw}{dt} - t\cos(tw)\frac{dw}{dt} &= w\cos(tw) - 2tw^3 - 9 \\
\frac{dw}{dt}[3t^2w^2 - t\cos(tw)] &= w\cos(tw) - 2tw^3 - 9 \\
\frac{dw}{dt} &= \frac{w\cos(tw) - 2tw^3 - 9}{3t^2w^2 - t\cos(tw)}.
\end{aligned}$$

[6] 2. We differentiate both sides of the equation with respect to x, giving

$$\begin{aligned}
[(1-y^2)(y+1)^2]' &= [(x^2-1)^2]' \\
[1-y^2]'(y+1)^2 + [(y+1)^2]'(1-y^2) &= 2(x^2-1)[x^2-1]' \\
(-2yy')(y+1)^2 + 2(y+1)y'(1-y^2) &= 2(x^2-1)(2x) \\
y'[-2y(y+1)^2 + 2(y+1)(1-y^2)] &= 4x(x^2-1) \\
y' &= -\frac{2x(x^2-1)}{y(y+1)^2 + (y+1)(1-y^2)} \\
&= -\frac{2x(x^2-1)}{(y+1)(2y^2+y-1)} \\
&= \frac{2x(x^2-1)}{1-3y^2-2y^3}.
\end{aligned}$$

At $\left(\sqrt{2}, 0\right)$,

$$y' = \frac{2\sqrt{2}(2-1)}{1-3(0^2)-2(0^3)} = 2\sqrt{2}.$$

Thus the equation of the tangent line is given by

$$y - 0 = 2\sqrt{2}\left(x - \sqrt{2}\right) \quad \Longrightarrow \quad y = 2\sqrt{2}x - 4.$$

[4] 3. (a) First we rewrite the given function using the properties of logarithms:

$$\begin{aligned} y &= \ln(x^7) - \ln\left(\sqrt{x\sec(x)}\right) \\ &= 7\ln(x) - \frac{1}{2}\ln(x\sec(x)) \\ &= 7\ln(x) - \frac{1}{2}\ln(x) - \frac{1}{2}\ln(\sec(x)) \\ &= \frac{13}{2}\ln(x) - \frac{1}{2}\ln(\sec(x)). \end{aligned}$$

Now we differentiate:

$$\begin{aligned} y' &= \frac{13}{2}\cdot\frac{1}{x} - \frac{1}{2}\cdot\frac{1}{\sec(x)}\cdot\sec(x)\tan(x) \\ &= \frac{13}{2x} - \frac{1}{2}\tan(x). \end{aligned}$$

[5] (b) We must use logarithmic differentiation. We take the logarithm of both sides, and get

$$\begin{aligned} \ln(y) &= \ln\left([\tan(3x)]^{\sqrt{x}}\right) \\ &= \sqrt{x}\ln(\tan(3x)). \end{aligned}$$

Now we differentiate implicitly, using the Product Rule and then the Chain Rule on the righthand side:

$$\begin{aligned} \frac{d}{dx}[\ln(y)] &= \frac{d}{dx}[\sqrt{x}\ln(\tan(3x))] \\ \frac{1}{y}\cdot\frac{dy}{dx} &= \frac{d}{dx}[\sqrt{x}]\ln(\tan(3x)) + \frac{d}{dx}[\ln(\tan(3x))]\sqrt{x} \\ &= \frac{1}{2\sqrt{x}}\ln(\tan(3x)) + \frac{1}{\tan(3x)}\cdot\frac{d}{dx}[\tan(3x)]\cdot\sqrt{x} \\ &= \frac{\ln(\tan(3x))}{2\sqrt{x}} + \frac{1}{\tan(3x)}\cdot\sec^2(3x)\cdot 3\cdot\sqrt{x} \\ &= \frac{\ln(\tan(3x))}{2\sqrt{x}} + \frac{3\sqrt{x}\sec^2(3x)}{\tan(3x)} \\ \frac{dy}{dx} &= y\left[\frac{\ln(\tan(3x))}{2\sqrt{x}} + \frac{3\sqrt{x}\sec^2(3x)}{\tan(3x)}\right] \\ &= [\tan(3x)]^{\sqrt{x}}\left[\frac{\ln(\tan(3x))}{2\sqrt{x}} + \frac{3\sqrt{x}\sec^2(3x)}{\tan(3x)}\right] \\ &= [\tan(3x)]^{\sqrt{x}}\left[\frac{\ln(\tan(3x))}{2\sqrt{x}} + \frac{3\sqrt{x}}{\sin(3x)\cos(3x)}\right]. \end{aligned}$$