MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 5 **MATHEMATICS 1000** FALL 2025

SOLUTIONS

[4] 1. (a) We use the Quotient Rule, followed by the Product Rule:

$$\begin{aligned} g'(t) &= \frac{(t^2e^t)'(t^2+e^t) - (t^2+e^t)'(t^2e^t)}{(t^2+e^t)^2} \\ &= \frac{[(t^2)'e^t + (e^t)'t^2](t^2+e^t) - (2t+e^t)(t^2e^t)}{(t^2+e^t)^2} \\ &= \frac{(2te^t + t^2e^t)(t^2+e^t) - (2t+e^t)(t^2e^t)}{(t^2+e^t)^2} \\ &= \frac{t^4e^t + 2te^{2t}}{(t^2+e^t)^2}. \end{aligned}$$

[4] (b) We use the Product Rule twice:

$$\begin{aligned} y' &= 2^x[\sin(x)\cos(x)]' + (2^x)'\sin(x)\cos(x) \\ &= 2^x[[\sin(x)]'\cos(x) + [\cos(x)]'\sin(x)] + 2^x\ln(2)\sin(x)\cos(x) \\ &= 2^x[[\cos(x)]\cos(x) + [-\sin(x)]\sin(x)] + 2^x\ln(2)\sin(x)\cos(x) \\ &= 2^x\cos^2(x) - 2^x\sin^2(x) + 2^x\ln(2)\sin(x)\cos(x). \end{aligned}$$

[4] (c) We use the Chain Rule, followed by the Product Rule:

$$\begin{aligned} \frac{dy}{dx} &= e^{x\cot(x)} \cdot \frac{d}{dx}[x\cot(x)] \\ &= e^{x\cot(x)} \cdot \left[\frac{d}{dx}[x]\cot(x) + \frac{d}{dx}[\cot(x)]x\right] \\ &= e^{x\cot(x)} \cdot \left[1\cdot\cot(x) + [-\csc^2(x)]x\right] \\ &= e^{x\cot(x)} \cdot \left[\cot(x) - x\csc^2(x)\right] \\ &= e^{x\cot(x)}\cot(x) - xe^{x\cot(x)}\csc^2(x). \end{aligned}$$

[4] (d) We use the Chain Rule twice:

$$\begin{aligned} \frac{d}{dx}[f(x)] &= 5\sec^4(x^3) \cdot \frac{d}{dx}[\sec(x^3)] \\ &= 5\sec^4(x^3)\cdot\sec(x^3)\tan(x^3)\cdot\frac{d}{dx}[x^3] \\ &= 5\sec^4(x^3)\cdot\sec(x^3)\tan(x^3)\cdot(3x^2) \\ &= 15x^2\sec^5(x^3)\tan(x^3). \end{aligned}$$

[4] 2. We are told that $s(0) = 0$, and since $s(0) = D$, we immediately have $D = 0$ and therefore

$$s(t) = At^3 + Bt^2 + Ct.$$

We also know that $v(0) = 0$, and

$$v(t) = s'(t) = A(3t^2) + B(2t) + C(1) = 3At^2 + 2Bt + C.$$

Thus $v(0) = C$ and so $C = 0,$ which means that

$$s(t) = At^3 + Bt^2.$$

Finally, we are given that $s(2) = s(6) = -36$. This means that

$$8A + 4B = -36 \quad \text{and} \quad 216A + 36B = -36.$$

Solving this system of equations gives $A = 2$ and $B = -13.$