## MEMORIAL UNIVERSITY OF NEWFOUNDLAND

## DEPARTMENT OF MATHEMATICS AND STATISTICS

Assignment 4

## MATHEMATICS 1000

**FALL 2025** 

## **SOLUTIONS**

[6] 1. By the limit definition,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3(x+h)}{x+h+2} - \frac{3x}{x+2}}{h}$$

$$= \lim_{h \to 0} \frac{1}{h} \cdot \left(\frac{3(x+h)(x+2)}{(x+2)(x+h+2)} - \frac{3x(x+h+2)}{(x+2)(x+h+2)}\right)$$

$$= \lim_{h \to 0} \frac{1}{h} \cdot \frac{(3x^2 + 6x + 3xh + 6h) - (3x^2 + 3xh + 6x)}{(x+2)(x+h+2)}$$

$$= \lim_{h \to 0} \frac{6h}{h(x+2)(x+h+2)}$$

$$= \lim_{h \to 0} \frac{6}{(x+2)(x+h+2)}$$

$$= \frac{6}{(x+2)^2}.$$

[4] 2. (a) By the limit definition of the derivative,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[(x+h)^3 - 5(x+h) + 1] - [x^3 - 5x + 1]}{h}$$

$$= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 5x - 5h + 1 - x^3 + 5x - 1}{h}$$

$$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3 - 5h}{h}$$

$$= \lim_{h \to 0} \frac{h(3x^2 + 3xh + h^2 - 5)}{h}$$

$$= \lim_{h \to 0} (3x^2 + 3xh + h^2 - 5)$$

$$= 3x^2 - 5.$$

[2] (b) From part (a), the slope of the tangent line at x = -2 is y' = 7. Furthermore, the y-coordinate of the point at x = -2 is y = 3, so (-2,3) is a point on the tangent line. Then we know that the slope-intercept form of the equation of the tangent line y = 7x + b where

$$3 = 7(-2) + b$$
$$17 = b.$$

Hence the equation of the tangent line is y = 7x + 17.

[4] 3. (a) We use the alternative definition of the derivative at x = 1,

$$f'(1) = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{f(x) - 3}{x - 1}.$$

Since f(x) changes definition at x = 1, we need to evaluate the one-sided limits. From the left,

$$\lim_{x \to 1^{-}} \frac{f(x) - 3}{x - 1} = \lim_{x \to 1^{-}} \frac{(4x - x^{2}) - 3}{x - 1} = \lim_{x \to 1^{-}} \frac{-(x - 3)(x - 1)}{x - 1} = \lim_{x \to 1^{-}} -(x - 3) = 2.$$

From the right,

$$\lim_{x \to 1^+} \frac{f(x) - 3}{x - 1} = \lim_{x \to 1^+} \frac{3x^2 - 3}{x - 1} = \lim_{x \to 1^+} \frac{3(x - 1)(x + 1)}{x - 1} = \lim_{x \to 1^+} 3(x + 1) = 6.$$

Since the one-sided limits differ,  $f'(1) = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}$  does not exist, and therefore f(x) is not differentiable at x = 1.

[4] (b) Again, we use the alternative definition of the derivative at x = 1,

$$g'(1) = \lim_{x \to 1} \frac{g(x) - g(1)}{x - 1} = \lim_{x \to 1} \frac{g(x) - 3}{x - 1}.$$

Since g(x) changes definition at x = 1, we need to evaluate the one-sided limits. From the left, we again have

$$\lim_{x \to 1^{-}} \frac{g(x) - 3}{x - 1} = \lim_{x \to 1^{-}} \frac{(4x - x^{2}) - 3}{x - 1} = \lim_{x \to 1^{-}} \frac{-(x - 3)(x - 1)}{x - 1} = \lim_{x \to 1^{-}} -(x - 3) = 2.$$

But this time, from the right, we obtain

$$\lim_{x \to 1^+} \frac{g(x) - 3}{x - 1} = \lim_{x \to 1^+} \frac{(x^2 + 2) - 3}{x - 1} = \lim_{x \to 1^+} \frac{(x - 1)(x + 1)}{x - 1} = \lim_{x \to 1^+} (x + 1) = 2.$$

Now the one-sided limits agree, the limit exists and therefore g(x) is differentiable at x = 1.