MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 4 | **MATH 1000** | FALL 2023

SOLUTIONS

[4] 1. Note that

$$f(x+h) = \frac{2(x+h)}{(x+h)+5} = \frac{2x+2h}{x+h+5},$$

so

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{\frac{2x+2h}{x+h+5} - \frac{2x}{x+5}}{h} \\
&= \lim_{h\to 0} \frac{(2x+2h)(x+5) - 2x(x+h+5)}{(x+5)(x+h+5)} \cdot \frac{1}{h} \\
&= \lim_{h\to 0} \frac{2x^2+2xh+10x+10h-2x^2-2xh-10x}{h(x+5)(x+h+5)} \\
&= \lim_{h\to 0} \frac{10h}{h(x+5)(x+h+5)} \\
&= \lim_{h\to 0} \frac{10}{(x+5)(x+h+5)} \\
&= \frac{10}{(x+5)^2}.
\end{aligned}$$

[4] 2. (a) Here,

$$f(x+h) = 5(x+h) - (x+h)^3 = 5x+5h-x^3-3x^2h-3xh^2-h^3$$

so

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{(5x+5h-x^3-3x^2h-3xh^2-h^3)-(5x-x^3)}{h} \\
&= \lim_{h\to 0} \frac{5h-3x^2h-3xh^2-h^3}{h} \\
&= \lim_{h\to 0} (5-3x^2-3xh-h^2) \\
&= 5-3x^2.
\end{aligned}$$

[2] (b) From part (a), the slope of the tangent line at $x = -1$ is $f'(-1) = 2$. Furthermore, the y-coordinate of the point at $x = -1$ is $f(-1) = -4$. Thus the tangent line has the form

$$y = 2x + b$$

where

$$\begin{aligned} -4 &= 2(-1) + b \\ -2 &= b. \end{aligned}$$

The equation of the tangent line is therefore $y = 2x - 2.$

[5] 3. (a) We use the alternate definition of the derivative:

$$f'(3) = \lim_{x \to 3} \frac{f(x) - f(3)}{x - 3} = \lim_{x \to 3} \frac{f(x) - 15}{x - 3}.$$

From the right, this limit becomes

$$\lim_{x \to 3^+} \frac{(2x^2 - 3) - 15}{x - 3} = \lim_{x \to 3^+} \frac{2x^2 - 18}{x - 3} = \lim_{x \to 3^+} \frac{2(x-3)(x+3)}{x - 3} = \lim_{x \to 3^+} 2(x+3) = 12.$$

However, from the left we have

$$\lim_{x \to 3^-} \frac{(x^2 + 2x) - 15}{x - 3} = \lim_{x \to 3^-} \frac{(x-3)(x+5)}{x - 3} = \lim_{x \to 3^-} (x+5) = 8.$$

Since the one-sided limits are not equal, the limit does not exist, and so $f(x)$ is not differentiable at $x = 3$.

[5] (b) The alternative definition of the derivatives indicates that

$$g'(3) = \lim_{x \to 3} \frac{g(x) - g(3)}{x - 3} = \lim_{x \to 3} \frac{g(x) - 27}{x - 3}.$$

From the right this becomes

$$\lim_{x \to 3^+} \frac{(2x^2 + 9) - 27}{x - 3} = \lim_{x \to 3^+} \frac{2x^2 - 18}{x - 3} = \lim_{x \to 3^+} \frac{2(x-3)(x+3)}{x - 3} = \lim_{x \to 3^+} 2(x+3) = 12.$$

From the left we have

$$\lim_{x \to 3^-} \frac{(x^2 + 6x) - 27}{x - 3} = \lim_{x \to 3^-} \frac{(x-3)(x+9)}{x - 3} = \lim_{x \to 3^-} (x+9) = 12$$

as well. Since the one-sided limits agree, we conclude that $g'(3) = 12$ and so $g(x)$ is differentiable at $x = 3$.