

# MEMORIAL UNIVERSITY OF NEWFOUNDLAND

## DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 4.6

Math 1000 Worksheet

FALL 2023

### SOLUTIONS

1. (a) This is a  $\frac{0}{0}$  indeterminate form:

$$\lim_{x \rightarrow 0} \frac{6^x - 2^x}{x} \stackrel{\text{H}}{=} \lim_{x \rightarrow 0} \frac{6^x \ln(6) - 2^x \ln(2)}{1} = \ln(6) - \ln(2) = \ln(3).$$

(b) This is a  $\frac{0}{0}$  indeterminate form:

$$\begin{aligned} \lim_{x \rightarrow 0^+} \frac{1 - \cos(\sqrt{x})}{x} &\stackrel{\text{H}}{=} \lim_{x \rightarrow 0^+} \frac{\sin(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}}{1} = \lim_{x \rightarrow 0^+} \frac{\sin(\sqrt{x})}{2\sqrt{x}} \\ &\stackrel{\text{H}}{=} \lim_{x \rightarrow 0^+} \frac{\cos(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}}{\frac{1}{\sqrt{x}}} = \lim_{x \rightarrow 0^+} \frac{\cos(\sqrt{x})}{2} = \frac{1}{2}. \end{aligned}$$

(c) This is a  $\frac{0}{0}$  indeterminate form:

$$\lim_{x \rightarrow 0} \frac{\sin(mx)}{\sin(nx)} \stackrel{\text{H}}{=} \lim_{x \rightarrow 0} \frac{m \cos(mx)}{n \cos(nx)} = \frac{m \cdot 1}{n \cdot 1} = \frac{m}{n}.$$

(d) This is an  $\frac{\infty}{\infty}$  indeterminate form:

$$\lim_{x \rightarrow \infty} \frac{\ln(1 + e^{2x})}{x} \stackrel{\text{H}}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{1+e^{2x}} \cdot 2e^{2x}}{1} = \lim_{x \rightarrow \infty} \frac{2e^{2x}}{1 + e^{2x}} \stackrel{\text{H}}{=} \lim_{x \rightarrow \infty} \frac{4e^{2x}}{2e^{2x}} = \lim_{x \rightarrow \infty} 2 = 2.$$

(e) This is an  $\frac{\infty}{\infty}$  indeterminate form:

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{[\ln(x)]^3}{x^2} &\stackrel{\text{H}}{=} \lim_{x \rightarrow \infty} \frac{3[\ln(x)]^2 \cdot \frac{1}{x}}{2x} \\ &= \lim_{x \rightarrow \infty} \frac{3[\ln(x)]^2}{2x^2} \\ &\stackrel{\text{H}}{=} \lim_{x \rightarrow \infty} \frac{6 \ln(x) \cdot \frac{1}{x}}{4x} \\ &= \lim_{x \rightarrow \infty} \frac{3 \ln(x)}{2x^2} \\ &\stackrel{\text{H}}{=} \lim_{x \rightarrow \infty} \frac{3 \cdot \frac{1}{x}}{4x} \\ &= \lim_{x \rightarrow \infty} \frac{3}{4x^2} \\ &= 0. \end{aligned}$$

(f) This is an  $\frac{\infty}{\infty}$  indeterminate form:

$$\lim_{x \rightarrow \infty} \frac{x^2 + 1}{x \ln(x)} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{2x}{x \left(\frac{1}{x}\right) + \ln(x)} = \lim_{x \rightarrow \infty} \frac{2x}{1 + \ln(x)} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{2}{\frac{1}{x}} = \lim_{x \rightarrow \infty} 2x = \infty.$$

(g) This is an  $\infty \cdot 0$  indeterminate form:

$$\lim_{x \rightarrow \frac{\pi}{2}^-} \sec(7x) \cos(3x) = \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\cos(3x)}{\cos(7x)} \stackrel{H}{=} \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{-3 \sin(3x)}{-7 \sin(7x)} = \frac{-3(-1)}{-7(-1)} = \frac{3}{7}.$$

(h) This is an  $\infty - \infty$  indeterminate form:

$$\begin{aligned} \lim_{x \rightarrow 1} \left( \frac{1}{\ln(x)} - \frac{1}{x-1} \right) &= \lim_{x \rightarrow 1} \frac{x-1 - \ln(x)}{(x-1)\ln(x)} \\ &\stackrel{H}{=} \lim_{x \rightarrow 1} \frac{1 - \frac{1}{x}}{(x-1) \cdot \frac{1}{x} + \ln(x)} \\ &= \lim_{x \rightarrow 1} \frac{x-1}{x-1 + x \ln(x)} \\ &\stackrel{H}{=} \lim_{x \rightarrow 1} \frac{1}{1 + \ln(x) + x \cdot \frac{1}{x}} \\ &= \lim_{x \rightarrow 1} \frac{1}{2 + \ln(x)} \\ &= \frac{1}{2}. \end{aligned}$$

(i) This is a  $0^0$  indeterminate form. Let  $y = \sin(x)^{\tan(x)}$  so  $\ln(y) = \tan(x) \ln(\sin(x))$ . Then

$$\begin{aligned} \lim_{x \rightarrow 0^+} \tan(x) \ln(\sin(x)) &= \lim_{x \rightarrow 0^+} \frac{\ln(\sin(x))}{\cot(x)} \stackrel{H}{=} \lim_{x \rightarrow 0^+} \frac{\frac{1}{\sin(x)} [\cos(x)]}{-\csc^2(x)} \\ &= \lim_{x \rightarrow 0^+} [-\sin(x) \cos(x)] = 0. \end{aligned}$$

Thus  $\lim_{x \rightarrow 0^+} (\sin(x))^{\tan(x)} = e^0 = 1.$

(j) This is an  $\infty^0$  indeterminate form. Let  $y = (x + e^x)^{\frac{1}{x}}$  so  $\ln(y) = \frac{1}{x} \cdot \ln(x + e^x)$ . Then

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{1}{x} \cdot \ln(x + e^x) &= \lim_{x \rightarrow \infty} \frac{\ln(x + e^x)}{x} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{x+e^x} \cdot (1 + e^x)}{1} = \lim_{x \rightarrow \infty} \frac{1 + e^x}{x + e^x} \\ &\stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{e^x}{1 + e^x} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{e^x}{e^x} = \lim_{x \rightarrow \infty} 1 = 1. \end{aligned}$$

Thus  $\lim_{x \rightarrow \infty} (x + e^x)^{\frac{1}{x}} = e^1 = e.$

(k) This is a  $1^\infty$  indeterminate form. Let  $y = \cos(3x)^{\frac{5}{x}}$  so  $\ln(y) = \frac{5}{x} \ln(\cos(3x))$ . Then

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{5}{x} \ln(\cos(3x)) &= \lim_{x \rightarrow 0} \frac{5 \ln(\cos(3x))}{x} \stackrel{\text{H}}{=} \lim_{x \rightarrow 0} \frac{5 \cdot \frac{1}{\cos(3x)} \cdot [-3 \sin(3x)]}{1} \\ &= \lim_{x \rightarrow 0} [-15 \tan(3x)] = 0.\end{aligned}$$

Thus  $\lim_{x \rightarrow 0} (\cos(3x))^{\frac{5}{x}} = e^0 = 1$ .

(ℓ) This is a  $1^\infty$  indeterminate form. Let  $y = \left(1 + \frac{a}{x}\right)^{bx}$  so  $\ln(y) = bx \ln\left(1 + \frac{a}{x}\right)$ . Then

$$\lim_{x \rightarrow \infty} bx \ln\left(1 + \frac{a}{x}\right) = \lim_{x \rightarrow \infty} \frac{b \ln\left(1 + \frac{a}{x}\right)}{\frac{1}{x}} \stackrel{\text{H}}{=} \lim_{x \rightarrow \infty} \frac{\frac{b}{1+\frac{a}{x}} \cdot \left(-\frac{a}{x^2}\right)}{-\frac{1}{x^2}} = \lim_{x \rightarrow \infty} \frac{ab}{1 + \frac{a}{x}} = ab.$$

Thus  $\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x}\right)^{bx} = e^{ab}$ .