

SOLUTIONS

[5] 1. (a) We use the Chain Rule twice:

$$\begin{aligned}y' &= 4 \tan^3(e^x) \cdot [\tan(e^x)]' \\&= 4 \tan^3(e^x) \cdot \sec^2(e^x) \cdot [e^x]' \\&= 4 \tan^3(e^x) \cdot \sec^2(e^x) \cdot e^x \\&= 4e^x \tan^3(e^x) \sec^2(e^x).\end{aligned}$$

[5] (b) We use the Product Rule, followed by the Chain Rule:

$$\begin{aligned}y' &= [x^4]' \tan(e^x) + x^4 [\tan(e^x)]' \\&= 4x^3 \tan(e^x) + x^4 \cdot \sec^2(e^x) \cdot [e^x]' \\&= 4x^3 \tan(e^x) + x^4 \cdot \sec^2(e^x) \cdot e^x \\&= 4x^3 \tan(e^x) + x^4 e^x \sec^2(e^x).\end{aligned}$$

[5] (c) We use the Chain Rule, followed by the Product Rule:

$$\begin{aligned}y' &= \sec^2(x^4 e^x) \cdot [x^4 e^x]' \\&= \sec^2(x^4 e^x) \cdot ([x^4]' e^x + x^4 [e^x]') \\&= \sec^2(x^4 e^x) \cdot (4x^3 e^x + x^4 e^x) \\&= 4x^3 e^x \sec^2(x^4 e^x) + x^4 e^x \sec^2(x^4 e^x).\end{aligned}$$

[5] (d) We use the Product Rule twice:

$$\begin{aligned}f'(x) &= [x^{-7}]' 7^x \sec(x) + x^{-7} [7^x \sec(x)]' \\&= -7x^{-8} 7^x \sec(x) + x^{-7} ([7^x]' \sec(x) + 7^x [\sec(x)]') \\&= -7x^{-8} 7^x \sec(x) + x^{-7} (7^x \ln(7) \sec(x) + 7^x \sec(x) \tan(x)) \\&= -7x^{-8} 7^x \sec(x) + x^{-7} 7^x \ln(7) \sec(x) + x^{-7} 7^x \sec(x) \tan(x).\end{aligned}$$

[5] (e) We use the Quotient Rule, followed by the Chain Rule:

$$\begin{aligned}f'(x) &= \frac{[\sin(5x)]'[\sin(5x) + 1] - \sin(5x)[\sin(5x) + 1]'}{[\sin(5x) + 1]^2} \\&= \frac{\cos(5x) \cdot [5x]'[\sin(5x) + 1] - \sin(5x) \cos(5x) \cdot [5x]'}{[\sin(5x) + 1]^2} \\&= \frac{\cos(5x) \cdot 5 \cdot [\sin(5x) + 1] - \sin(5x) \cos(5x) \cdot 5}{[\sin(5x) + 1]^2} \\&= \frac{5 \sin(5x) \cos(5x) + 5 \cos(5x) - 5 \sin(5x) \cos(5x)}{[\sin(5x) + 1]^2} \\&= \frac{5 \cos(5x)}{[\sin(5x) + 1]^2}.\end{aligned}$$

[5] (f) We use the Quotient Rule, followed by the Product Rule:

$$\begin{aligned}y' &= \frac{[x \cos(x)]'(x^2 - 4) - x \cos(x)[x^2 - 4]'}{(x^2 - 4)^2} \\&= \frac{([x]' \cos(x) + x[\cos(x)]')(x^2 - 4) - x \cos(x) \cdot 2x}{(x^2 - 4)^2} \\&= \frac{(1 \cdot \cos(x) - x \sin(x))(x^2 - 4) - 2x^2 \cos(x)}{(x^2 - 4)^2} \\&= \frac{(\cos(x) - x \sin(x))(x^2 - 4) - 2x^2 \cos(x)}{(x^2 - 4)^2}.\end{aligned}$$

Optionally, we could further write

$$\begin{aligned}y' &= \frac{x^2 \cos(x) - x^3 \sin(x) - 4 \cos(x) + 4x \sin(x) - 2x^2 \cos(x)}{(x^2 - 4)^2} \\&= \frac{4x \sin(x) - x^3 \sin(x) - 4 \cos(x) - x^2 \cos(x)}{(x^2 - 4)^2}.\end{aligned}$$

[5] 2. We use implicit differentiation, applying the Product Rule to the lefthand side:

$$\begin{aligned}\frac{d}{dx}[x^3y^3] &= \frac{d}{dx}[6x + 2y] \\ \frac{d}{dx}[x^3]y^3 + x^3 \cdot \frac{d}{dx}[y^3] &= 6 + 2\frac{dy}{dx} \\ 3x^2y^3 + x^3 \cdot 3y^2\frac{dy}{dx} &= 6 + 2\frac{dy}{dx} \\ 3x^3y^2\frac{dy}{dx} - 2\frac{dy}{dx} &= 6 - 3x^2y^3 \\ \frac{dy}{dx}(3x^3y^2 - 2) &= 6 - 3x^2y^3 \\ \frac{dy}{dx} &= \frac{6 - 3x^2y^3}{3x^3y^2 - 2}.\end{aligned}$$

[5] 3. We have

$$\begin{aligned}[\sin(x)]' &= \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{[\sin(x)\cos(h) + \cos(x)\sin(h)] - \sin(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin(x)\cos(h) - \sin(x)}{h} + \lim_{h \rightarrow 0} \frac{\cos(x)\sin(h)}{h} \\ &= \sin(x) \lim_{h \rightarrow 0} \frac{\cos(h) - 1}{h} + \cos(x) \lim_{h \rightarrow 0} \frac{\sin(h)}{h} \\ &= \sin(x) \cdot 0 + \cos(x) \cdot 1 \\ &= \cos(x).\end{aligned}$$