MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Assignment 3

MATHEMATICS 1000

Fall 2025

SOLUTIONS

[5] 1. Since this is a quasirational function, we need to evaluate both limits at infinity separately. In each case, note that the effective highest power in the denominator is x^1 , since the highest power outside the radical is x, while the highest power inside the square root is x^2 , which we treat as $x^{\frac{2}{2}} = x^1$.

For $x \to \infty$, we recall that $x = \sqrt{x^2}$ and write

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{3 - 5x}{2x - \sqrt{4x^2 + 5}} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x \to \infty} \frac{\frac{3}{x} - 5}{2 - \frac{1}{x}\sqrt{4x^2 + 5}}$$

$$= \lim_{x \to \infty} \frac{\frac{\frac{3}{x} - 5}{2 - \frac{1}{\sqrt{x^2}}\sqrt{4x^2 + 5}}$$

$$= \lim_{x \to \infty} \frac{\frac{\frac{3}{x} - 5}{2 - \sqrt{4 + \frac{5}{x^2}}}$$

$$= \frac{0 - 5}{2 - \sqrt{4 + 0}}$$

$$= \frac{-5}{0}.$$

Since we have now obtained a $\frac{K}{0}$ form, this means that the limit does not exist.

When $x \to -\infty$, we know that $x = -\sqrt{x^2}$, so now

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{3 - 5x}{2x - \sqrt{4x^2 + 5}} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x \to -\infty} \frac{\frac{3}{x} - 5}{2 - \frac{1}{x}\sqrt{4x^2 + 5}}$$

$$= \lim_{x \to -\infty} \frac{\frac{3}{x} - 5}{2 - \frac{1}{-\sqrt{x^2}}\sqrt{4x^2 + 5}}$$

$$= \lim_{x \to -\infty} \frac{\frac{3}{x} - 5}{2 + \sqrt{4 + \frac{5}{x^2}}}$$

$$= \frac{0 - 5}{2 + \sqrt{4 + 0}}$$

$$= -\frac{5}{4}.$$

Consequently, the only horizontal asymptote to the graph of this function is the line y =

[5] 2. First observe that f(-4) = k - 6, which is defined for all values of k. Next, we need to take the limit as $x \to -4$. Note that f(x) is defined the same way both to the left and to the right of x = -4, so there's no need to consider the one-sided limits. Instead, we just have

$$\lim_{x \to -4} f(x) = \lim_{x \to -4} \frac{kx}{3k+x} = \frac{-4k}{3k-4}.$$

This limit will exist as long as $3k-4\neq 0$, that is, for $k\neq \frac{4}{3}$. Finally, we must have

$$f(-4) = \lim_{x \to -4} f(x)$$

$$k - 6 = \frac{-4k}{3k - 4}$$

$$\frac{(k - 6)(3k - 4)}{3k - 4} = \frac{-4k}{3k - 4}$$

$$3k^2 - 22k + 24 = -4k$$

$$3k^2 - 18k + 24 = 0$$

$$3(k^2 - 6k + 8) = 0$$

$$3(k - 4)(k - 2) = 0$$

which is true only when k=4 or k=2. Hence these are the only values of k for which f(x) is continuous at x=-4.

[2] 3. (a) We have f(0) = 6. The one-sided limits are

$$\lim_{x \to 0^{-}} f(x) = \lim_{x \to 0^{-}} \frac{x+6}{2x+3} = \frac{6}{3} = 2$$

and

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (x^2 - 5x + 6) = 6.$$

Since the one-sided limits disagree, $\lim_{x\to 0} f(x)$ does not exist, and therefore x=0 is a non-removable discontinuity.

[2] (b) We have f(3) = 0. The one-sided limits are

$$\lim_{x \to 3^{-}} f(x) = \lim_{x \to 3^{-}} (x^{2} - 5x + 6) = 0$$

and

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} \frac{x^2 - 7x + 12}{x^2 - 5x + 4} = \frac{0}{-2} = 0$$

so

$$\lim_{x \to 3} f(x) = 0 = f(3).$$

Hence f(x) is continuous at x = 3.

[6] (c) Now we consider any values of x that would make any part of the definition of f(x) undefined.

For $\frac{x+6}{2x+3}$, the denominator will be zero if 2x+3=0, so $x=-\frac{3}{2}$. This is in the appropriate interval (x<0) so the function will be discontinuous there. Furthermore, because direct substitution results in a $\frac{9}{2}$ form, the limit as $x\to -\frac{3}{2}$ does not exist, and therefore $x=-\frac{3}{2}$ is a non-removable discontinuity.

For $4x^2 - 5x - 6$ there are no possible discontinuities because this part of the definition is a polynomial.

For $\frac{x^2-7x+12}{x^2-5x+4}$, the denominator will be zero if $x^2-5x+4=(x-4)(x-1)=0$, so x=4 or x=1. We can omit x=1 because it does not lie in the appropriate interval $(x \ge 3)$, but x=4 is a discontinuity. Direct substitution results in a $\frac{0}{0}$ form, so we take the limit:

$$\lim_{x \to 4} \frac{x^2 - 7x + 12}{x^2 - 5x + 4} = \lim_{x \to 4} \frac{(x - 4)(x - 3)}{(x - 4)(x - 1)} = \lim_{x \to 4} \frac{x - 3}{x - 1} = \frac{1}{3}.$$

Since the limit exists, x = 3 is a removable discontinuity.