MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 3.7 | **Math 1000 Worksheet** | FALL 2023

SOLUTIONS

1. We compute each derivative in turn:

$$\begin{aligned}
f'(x) &= (x^2)'e^x + (e^x)'x^2 = 2xe^x + x^2e^x \\
f''(x) &= [(2x)'e^x + (e^x)'(2x)] + [(x^2)'e^x + (e^x)'x^2] \\
&= [2e^x + 2xe^x] + [2xe^x + x^2e^x] = 2e^x + 4xe^x + x^2e^x \\
f'''(x) &= 2(e^x)' + [(4x)'e^x + (e^x)'(4x)] + [(x^2)'e^x + (e^x)'x^2] \\
&= 2e^x + [4e^x + 4xe^x] + [2xe^x + x^2e^x] \\
&= 6e^x + 6xe^x + x^2e^x.
\end{aligned}$$

2. We have

$$\begin{aligned}
f'(x) &= (x)'\sin(x) + [\sin(x)]'x = (1)\sin(x) + \cos(x)x = \sin(x) + x\cos(x) \\
f''(x) &= [\sin(x)]' + (x)'\cos(x) + [\cos(x)]'x = \cos(x) + (1)\cos(x) + [-\sin(x)]x \\
&= 2\cos(x) - x\sin(x) \\
f'''(x) &= [2\cos(x)]' - (x)'\sin(x) - [\sin(x)]'x = -2\sin(x) - (1)\sin(x) - \cos(x)x \\
&= -3\sin(x) - x\cos(x) \\
f^{(4)}(x) &= [-3\sin(x)]' - (x)'\cos(x) - x[\cos(x)]' \\
&= -3\cos(x) - (1)\cos(x) - x[-\sin(x)] = -4\cos(x) + x\sin(x).
\end{aligned}$$

3. We use the Chain Rule to obtain the first derivative:

$$y' = \sec^2(x^2)\cdot(x^2)' = 2x\sec^2(x^2).$$

Next we need to use both the Chain Rule and the Product Rule to find the second derivative:

$$\begin{aligned}
y'' &= (2x)'\sec^2(x^2) + [\sec^2(x^2)]'(2x) \\
&= 2\sec^2(x^2) + 2\sec(x^2)\cdot[\sec(x^2)]'(2x) \\
&= 2\sec^2(x^2) + 2\sec(x^2)\cdot\sec(x^2)\tan(x^2)\cdot(x^2)'(2x) \\
&= 2\sec^2(x^2) + 2\sec(x^2)\cdot\sec(x^2)\tan(x^2)\cdot(2x)(2x) \\
&= 2\sec^2(x^2) + 8x^2\sec^2(x^2)\tan(x^2).
\end{aligned}$$

4. We begin by finding the first derivative:

$$\begin{aligned}\frac{d}{dx}[y] &= \frac{d}{dx}[2x - y^2] \\ \frac{dy}{dx} &= 2 - 2y\frac{dy}{dx} \\ \frac{dy}{dx} + 2y\frac{dy}{dx} &= 2 \\ \frac{dy}{dx}(1+2y) &= 2 \\ \frac{dy}{dx} &= \frac{2}{1+2y}.\end{aligned}$$

We differentiate again, substituting the expression for $\dfrac{dy}{dx}$ where possible:

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{2}{1+2y}\right] = \frac{0 - 2\cdot 2\frac{dy}{dx}}{(1+2y)^2} = \frac{-4\frac{dy}{dx}}{(1+2y)^2} = \frac{-4\left(\frac{2}{1+2y}\right)}{(1+2y)^2} = \frac{-8}{(1+2y)^3}.$$

5. We differentiate implicitly to find the first derivative:

$$\begin{aligned}\frac{d}{dx}\left[\sqrt{x} + \sqrt{y}\right] &= \frac{d}{dx}[2] \\ \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx} &= 0 \\ \frac{dy}{dx} &= -\left(\frac{y}{x}\right)^{\frac{1}{2}}.\end{aligned}$$

Differentiating implicitly a second time gives us

$$\begin{aligned}\frac{d^2y}{dx^2} &= -\frac{1}{2}\left(\frac{y}{x}\right)^{-\frac{1}{2}}\left(\frac{\frac{d}{dx}[y]x - \frac{d}{dx}[x]y}{x^2}\right) = -\frac{1}{2}\left(\frac{y}{x}\right)^{-\frac{1}{2}}\left(\frac{\left(\frac{dy}{dx}\right)x - (1)y}{x^2}\right) \\ &= -\frac{1}{2}\left(\frac{x}{y}\right)^{\frac{1}{2}}\left(\frac{-\left(\frac{y}{x}\right)^{\frac{1}{2}}x - y}{x^2}\right) = \frac{\sqrt{x}+\sqrt{y}}{2x^{\frac{3}{2}}}.\end{aligned}$$

But remember that $\sqrt{x} + \sqrt{y} = 2$, so this becomes

$$\frac{d^2y}{dx^2} = \frac{2}{2x^{\frac{3}{2}}} = x^{-\frac{3}{2}}.$$

6. We have

$$\begin{aligned} v(t) = s'(t) &= \frac{(49t-10)'(t+10)-(t+10)'(49t-10)}{(t+10)^2} \\ &= \frac{(49)(t+10)-(1)(49t-10)}{(t+10)^2} = \frac{500}{(t+10)^2} = \frac{500}{t^2+20t+100} \\ a(t) = s''(t) &= \frac{(500)'(t^2+20t+100)-(t^2+20t+100)'(500)}{(t^2+20t+100)^2} \\ &= \frac{0-(2t+20)(500)}{(t^2+20t+100)^2} = \frac{-1000(t+10)}{(t+10)^4} = -\frac{1000}{(t+10)^3}. \end{aligned}$$

Therefore,

$$v(0) = \frac{500}{100} = 5 \quad \text{and} \quad a(0) = \frac{-1000}{1000} = -1.$$

Hence the puppy's initial velocity is 5 cm/sec and its initial acceleration is -1 cm/sec^2.