MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 3.6 | **Math 1000 Worksheet** | FALL 2025

SOLUTIONS

1. (a) $\dfrac{dy}{dx} = \cosh(x^3) \cdot \dfrac{d}{dx}[x^3] = 3x^2\cosh(x^3)$

   (b) $\dfrac{dy}{dx} = 3\sinh^2(x) \cdot \dfrac{d}{dx}[\sinh(x)] = 3\sinh^2(x)\cosh(x)$

   (c) $f'(x) = \dfrac{[\cosh(x)]'\cos(x) - [\cos(x)]'\cosh(x)}{[\cos(x)]^2} = \dfrac{\sinh(x)\cos(x) - [-\sin(x)]\cosh(x)}{\cos^2(x)}$

   $$= \frac{\sinh(x)\cos(x) + \sin(x)\cosh(x)}{\cos^2(x)}$$

   (d) We must use logarithmic differentiation:

   $$\begin{aligned} \ln(y) &= \ln(x^{\cosh(x)}) = \cosh(x)\ln(x) \\ \frac{1}{y}\cdot\frac{dy}{dx} &= \sinh(x)\ln(x) + \cosh(x)\cdot\frac{1}{x} \\ \frac{dy}{dx} &= y\left[\sinh(x)\ln(x) + \frac{\cosh(x)}{x}\right] = x^{\cosh(x)}\left[\sinh(x)\ln(x) + \frac{\cosh(x)}{x}\right]. \end{aligned}$$

2. Since $\sinh(x) = \dfrac{e^x - e^{-x}}{2}$,

   $$\frac{d}{dx}[\sinh(x)] = \frac{d}{dx}\left[\frac{e^x - e^{-x}}{2}\right] = \frac{1}{2}[e^x - e^{-x}\cdot(-1)] = \frac{e^x + e^{-x}}{2} = \cosh(x).$$

3. We start from the righthand side:

   $$\begin{aligned} &\sinh(x)\cosh(y) + \cosh(x)\sinh(y) \\ &= \left(\frac{e^x - e^{-x}}{2}\right)\left(\frac{e^y + e^{-y}}{2}\right) + \left(\frac{e^x + e^{-x}}{2}\right)\left(\frac{e^y - e^{-y}}{2}\right) \\ &= \frac{e^xe^y - e^{-x}e^y + e^xe^{-y} - e^{-x}e^{-y}}{4} + \frac{e^xe^y + e^{-x}e^y - e^xe^{-y} - e^{-x}e^{-y}}{4} \\ &= \frac{2e^xe^y - 2e^{-x}e^{-y}}{4} = \frac{e^{x+y} - e^{-(x+y)}}{2} = \sinh(x+y). \end{aligned}$$