MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 3.6 **Math 1000 Worksheet** FALL 2023

SOLUTIONS

1. (a) $\dfrac{dy}{dx} = \cosh(x^3) \cdot \dfrac{d}{dx}[x^3] = 3x^2\cosh(x^3)$

   (b) $\dfrac{dy}{dx} = 3\sinh^2(x) \cdot \dfrac{d}{dx}[\sinh(x)] = 3\sinh^2(x)\cosh(x)$

   (c) $$\begin{aligned} f'(x) &= \frac{[\operatorname{sech}(x)]'\sec(x) - [\sec(x)]'\operatorname{sech}(x)}{[\sec(x)]^2} \\ &= \frac{-\operatorname{sech}(x)\tanh(x)\sec(x) - \sec(x)\tan(x)\operatorname{sech}(x)}{\sec^2(x)} \\ &= -\frac{\operatorname{sech}(x)\tanh(x) + \tan(x)\operatorname{sech}(x)}{\sec(x)} \end{aligned}$$

   (d) We must use logarithmic differentiation:

   $$\begin{aligned} \ln(y) &= \ln(x^{\tanh(x)}) = \tanh(x)\ln(x) \\ \frac{1}{y} \cdot \frac{dy}{dx} &= \operatorname{sech}^2(x)\ln(x) + \tanh(x) \cdot \frac{1}{x} \\ \frac{dy}{dx} &= y\left[\operatorname{sech}^2(x)\ln(x) + \frac{\tanh(x)}{x}\right] = x^{\tanh(x)}\left[\operatorname{sech}^2(x)\ln(x) + \frac{\tanh(x)}{x}\right]. \end{aligned}$$

2. Since $\sinh(x) = \dfrac{e^x - e^{-x}}{2}$,

   $$\frac{d}{dx}[\sinh(x)] = \frac{d}{dx}\left[\frac{e^x - e^{-x}}{2}\right] = \frac{1}{2}[e^x - e^{-x} \cdot (-1)] = \frac{e^x + e^{-x}}{2} = \cosh(x).$$

3. We start from the righthand side:

   $$\begin{aligned} &\sinh(x)\cosh(y) + \cosh(x)\sinh(y) \\ &= \left(\frac{e^x - e^{-x}}{2}\right)\left(\frac{e^y + e^{-y}}{2}\right) + \left(\frac{e^x + e^{-x}}{2}\right)\left(\frac{e^y - e^{-y}}{2}\right) \\ &= \frac{e^xe^y - e^{-x}e^y + e^xe^{-y} - e^{-x}e^{-y}}{4} + \frac{e^xe^y + e^{-x}e^y - e^xe^{-y} - e^{-x}e^{-y}}{4} \\ &= \frac{2e^xe^y - 2e^{-x}e^{-y}}{4} = \frac{e^{x+y} - e^{-(x+y)}}{2} = \sinh(x + y). \end{aligned}$$