## MEMORIAL UNIVERSITY OF NEWFOUNDLAND

## DEPARTMENT OF MATHEMATICS AND STATISTICS

Section 3.3

## Math 1000 Worksheet

Fall 2025

## SOLUTIONS

1. We differentiate both sides with respect to x, giving

$$\frac{d}{dx}[y^2] = \frac{d}{dx}[x^4 - x^6]$$

$$2y\frac{dy}{dx} = 4x^3 - 6x^5$$

$$\frac{dy}{dx} = \frac{4x^3 - 6x^5}{2y} = \frac{2x^3 - 3x^5}{y}.$$

2. We differentiate both sides with respect to x, giving

$$\frac{d}{dx}[2y^3 + 3y^2] = \frac{d}{dx}[(x^2 - 1)^2]$$

$$6y^2 \frac{dy}{dx} + 6y \frac{dy}{dx} = 2(x^2 - 1) \cdot 2x$$

$$\frac{dy}{dx}(6y^2 + 6y) = 4x(x^2 - 1)$$

$$\frac{dy}{dx} = \frac{2x(x^2 - 1)}{3y(y + 1)}.$$

3. We differentiate implicitly, using the Product Rule on the righthand side:

$$\frac{d}{dx}[9y^2] = \frac{d}{dx}[(y-1)^2(x^2+y^2)]]$$

$$18y\frac{dy}{dx} = 2(y-1)\frac{dy}{dx} \cdot (x^2+y^2) + \left(2x+2y\frac{dy}{dx}\right) \cdot (y-1)^2$$

$$-2x(y-1)^2 = \frac{dy}{dx}[2(y-1)(x^2+y^2) + 2y(y-1)^2 - 18y]$$

$$\frac{dy}{dx} = \frac{-2x(y-1)^2}{2(y-1)(x^2+y^2) + 2y(y-1)^2 - 18y}$$

$$= \frac{-x(y-1)^2}{x^2y - x^2 + 2y^3 - 3y^2 - 8y}.$$

4. We differentiate implicitly:

$$\frac{d}{dx} \left[ \frac{xy}{\pi} \right] = \frac{d}{dx} [\cos(x+y)]$$

$$\frac{1}{\pi} \left( \frac{d}{dx} [x] y + x \frac{d}{dx} [y] \right) = -\sin(x+y) \cdot \frac{d}{dx} [x+y]$$

$$\frac{1}{\pi} \left( y + x \frac{dy}{dx} \right) = -\sin(x+y) \left( 1 + \frac{dy}{dx} \right)$$

$$\frac{dy}{dx} \left[ \frac{x}{\pi} + \sin(x+y) \right] = -\frac{y}{\pi} - \sin(x+y)$$

$$\frac{dy}{dx} = \frac{-\frac{y}{\pi} - \sin(x+y)}{\frac{x}{\pi} + \sin(x+y)} = \frac{-y - \pi \sin(x+y)}{x + \pi \sin(x+y)}.$$

At the point  $(0, \frac{\pi}{2})$ ,

$$\frac{dy}{dx} = \frac{-\frac{\pi}{2} - \pi \sin\left(\frac{\pi}{2}\right)}{0 + \pi \sin\left(\frac{\pi}{2}\right)} = \frac{-\frac{3\pi}{2}}{\pi} = -\frac{3}{2}$$

so then the equation of the tangent line is

$$y - \frac{\pi}{2} = -\frac{3}{2}(x - 0) \implies y = -\frac{3}{2}x + \frac{\pi}{2}.$$

The slope of the normal line is  $\frac{2}{3}$ , the negative reciprocal of the slope of the tangent line. Hence the equation of the normal line is

$$y - \frac{\pi}{2} = \frac{2}{3}(x - 0) \implies y = \frac{2}{3}x + \frac{\pi}{2}.$$