MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Section 3.3	**Math 1000 Worksheet**	Fall 2023

SOLUTIONS

1. We differentiate both sides with respect to x, giving

$$\begin{aligned}\frac{d}{dx}[y^2] &= \frac{d}{dx}[x^4 - x^6]\\ 2y\frac{dy}{dx} &= 4x^3 - 6x^5\\ \frac{dy}{dx} &= \frac{4x^3-6x^5}{2y} = \frac{2x^3-3x^5}{y}.\end{aligned}$$

2. We differentiate both sides with respect to x, giving

$$\begin{aligned}\frac{d}{dx}[2y^3+3y^2] &= \frac{d}{dx}[(x^2-1)^2]\\ 6y^2\frac{dy}{dx} + 6y\frac{dy}{dx} &= 2(x^2-1)\cdot 2x\\ \frac{dy}{dx}(6y^2+6y) &= 4x(x^2-1)\\ \frac{dy}{dx} &= \frac{2x(x^2-1)}{3y(y+1)}.\end{aligned}$$

3. We differentiate implicitly, using the Product Rule on the righthand side:

$$\begin{aligned}\frac{d}{dx}[9y^2] &= \frac{d}{dx}[(y-1)^2(x^2+y^2)]]\\ 18y\frac{dy}{dx} &= 2(y-1)\frac{dy}{dx}\cdot(x^2+y^2) + \left(2x+2y\frac{dy}{dx}\right)\cdot(y-1)^2\\ -2x(y-1)^2 &= \frac{dy}{dx}[2(y-1)(x^2+y^2)+2y(y-1)^2-18y]\\ \frac{dy}{dx} &= \frac{-2x(y-1)^2}{2(y-1)(x^2+y^2)+2y(y-1)^2-18y}\\ &= \frac{-x(y-1)^2}{x^2y-x^2+2y^3-3y^2-8y}.\end{aligned}$$

4. We differentiate implicitly:

$$\begin{aligned}
\frac{d}{dx}\left[\frac{xy}{\pi}\right] &= \frac{d}{dx}[\cos(x+y)] \\
\frac{1}{\pi}\left(\frac{d}{dx}[x]y + x\frac{d}{dx}[y]\right) &= -\sin(x+y)\cdot\frac{d}{dx}[x+y] \\
\frac{1}{\pi}\left(y + x\frac{dy}{dx}\right) &= -\sin(x+y)\left(1+\frac{dy}{dx}\right) \\
\frac{dy}{dx}\left[\frac{x}{\pi}+\sin(x+y)\right] &= -\frac{y}{\pi} - \sin(x+y) \\
\frac{dy}{dx} &= \frac{-\frac{y}{\pi}-\sin(x+y)}{\frac{x}{\pi}+\sin(x+y)} = \frac{-y-\pi\sin(x+y)}{x+\pi\sin(x+y)}.
\end{aligned}$$

At the point $\left(0, \frac{\pi}{2}\right)$,

$$\frac{dy}{dx} = \frac{-\frac{\pi}{2}-\pi\sin\left(\frac{\pi}{2}\right)}{0+\pi\sin\left(\frac{\pi}{2}\right)} = \frac{-\frac{3\pi}{2}}{\pi} = -\frac{3}{2}$$

so then the equation of the tangent line is

$$y - \frac{\pi}{2} = -\frac{3}{2}(x-0) \quad\Longrightarrow\quad \boxed{y = -\frac{3}{2}x + \frac{\pi}{2}.}$$

The slope of the normal line is $\frac{2}{3}$, the negative reciprocal of the slope of the tangent line. Hence the equation of the normal line is

$$y - \frac{\pi}{2} = \frac{2}{3}(x-0) \quad\Longrightarrow\quad \boxed{y = \frac{2}{3}x + \frac{\pi}{2}.}$$