SOLUTIONS

1. (a) $\dfrac{dy}{dx} = \dfrac{1}{2}(4e^x + 2)^{-\frac{1}{2}} \cdot \dfrac{d}{dx}[4e^x + 2] = \dfrac{1}{2}(4e^x + 2)^{-\frac{1}{2}} \cdot 4e^x = \dfrac{2e^x}{\sqrt{4e^x + 2}}$

   (b) $y' = \cos(x^2 - 7x) \cdot (x^2 - 7x)' = \cos(x^2 - 7x) \cdot (2x - 7)$

   $= (2x - 7)\cos(x^2 - 7x)$

   (c) $f'(x) = e^{5^x} \cdot (5^x)' = e^{5^x} \cdot [5^x \ln(5)] = e^{5^x} 5^x \ln(5)$

   (d) We use the Chain Rule twice:

   $$\dfrac{dy}{dx} = -\sin(\cos(x)) \cdot \dfrac{d}{dx}[\cos(x)] = -\sin(\cos(x)) \cdot [-\sin(x)]$$

   $$= \sin(x)\sin(\cos(x)).$$

   (e) We use the Product Rule, then the Chain Rule for each of the resulting derivatives:

   $$\dfrac{d}{dx}[f(x)] = \dfrac{d}{dx}[(5 - 2x)^{-4}]e^{\sec(x)} + \dfrac{d}{dx}[e^{\sec(x)}](5 - 2x)^{-4}$$

   $$= -4(5 - 2x)^{-5} \cdot \dfrac{d}{dx}[5 - 2x]e^{\sec(x)} + e^{\sec(x)} \cdot \dfrac{d}{dx}[\sec(x)](5 - 2x)^{-4}$$

   $$= -4(5 - 2x)^{-5} \cdot (-2)e^{\sec(x)} + e^{\sec(x)} \cdot [\sec(x)\tan(x)](5 - 2x)^{-4}$$

   $$= 8(5 - 2x)^{-5}e^{\sec(x)} + (5 - 2x)^{-4}e^{\sec(x)}\sec(x)\tan(x).$$

   (f) We use the Chain Rule, then the Product Rule:

   $$f'(t) = \sec^2(t^2 e^t) \cdot (t^2 e^t)'$$

   $$= \sec^2(t^2 e^t) \cdot [(t^2)'e^t + (e^t)'t^2] = (2te^t + t^2 e^t)\sec^2(t^2 e^t).$$

   (g) First we use the Quotient Rule, then the Chain Rule:

   $$f'(x) = \dfrac{(x^2)'(x^3 - x)^6 - [(x^3 - x)^6]'x^2}{[(x^3 - x)^6]^2}$$

   $$= \dfrac{2x(x^3 - x)^6 - 6(x^3 - x)^5 \cdot (x^3 - x)'x^2}{[(x^3 - x)^{12}}$$

   $$= \dfrac{2x(x^3 - x)^6 - 6(x^3 - x)^5 \cdot (3x^2 - 1)x^2}{(x^3 - x)^{12}}$$

   $$= \dfrac{2x(x^3 - x) - 6x^2(3x^2 - 1)}{(x^3 - x)^7}$$

   $$= \dfrac{-16x^4 + 4x^2}{(x^3 - x)^7}.$$

(h) We use the Chain Rule three times:

$$h'(x) = \cos(\cos(\tan(x)))[\cos(\tan(x))]'$$
$$= \cos(\cos(\tan(x)))[\cos(\tan(x))]'$$
$$= -\cos(\cos(\tan(x)))\sin(\tan(x))[\tan(x)]'$$
$$= -\cos(\cos(\tan(x)))\sin(\tan(x))\sec^2(x).$$