

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

TEST 2

MATHEMATICS 1000-003

FALL 2025

SOLUTIONS

- [8] 1. By the limit definition of the derivative,

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\sqrt{4(x+h)+5} - \sqrt{4x+5}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\sqrt{4x+4h+5} - \sqrt{4x+5}}{h} \cdot \frac{\sqrt{4x+4h+5} + \sqrt{4x+5}}{\sqrt{4x+4h+5} + \sqrt{4x+5}} \\
 &= \lim_{h \rightarrow 0} \frac{(4x+4h+5) - (4x+5)}{h(\sqrt{4x+4h+5} + \sqrt{4x+5})} \\
 &= \lim_{h \rightarrow 0} \frac{4h}{h(\sqrt{4x+4h+5} + \sqrt{4x+5})} \\
 &= \lim_{h \rightarrow 0} \frac{4}{\sqrt{4x+4h+5} + \sqrt{4x+5}} \\
 &= \frac{4}{\sqrt{4x+5} + \sqrt{4x+5}} \\
 &= \frac{2}{\sqrt{4x+5}}.
 \end{aligned}$$

- [4] 2. One possibility is a function with a sharp corner (a cusp) at $x = 0$, such as $f(x) = |x|$. Another possibility is a function with a vertical tangent line at $x = 0$, such as $f(x) = \sqrt[3]{x}$.

- [5] 3. (a) We use the Chain Rule twice:

$$\begin{aligned}
 y' &= 3 \sin^2 \left(\frac{1}{x} \right) \cdot \left[\sin \left(\frac{1}{x} \right) \right]' \\
 &= 3 \sin^2 \left(\frac{1}{x} \right) \cdot \cos \left(\frac{1}{x} \right) \cdot \left[\frac{1}{x} \right]' \\
 &= 3 \sin^2 \left(\frac{1}{x} \right) \cdot \cos \left(\frac{1}{x} \right) \cdot \left(-\frac{1}{x^2} \right) \\
 &= \frac{-3 \sin^2 \left(\frac{1}{x} \right) \cos \left(\frac{1}{x} \right)}{x^2}.
 \end{aligned}$$

[4] (b) We use the Quotient Rule, follow by the Chain Rule:

$$\begin{aligned}
 y' &= \frac{[1 - e^{2x}]'(1 + e^{2x}) - (1 - e^{2x})[1 + e^{2x}]'}{(1 + e^{2x})^2} \\
 &= \frac{-e^{2x} \cdot [2x]' \cdot (1 + e^{2x}) - (1 - e^{2x}) \cdot e^{2x} \cdot [2x]'}{(1 + e^{2x})^2} \\
 &= \frac{-e^{2x} \cdot 2 \cdot (1 + e^{2x}) - (1 - e^{2x}) \cdot e^{2x} \cdot 2}{(1 + e^{2x})^2} \\
 &= \frac{-2e^{2x} - 2e^{4x} - 2e^{2x} + 2e^{4x}}{(1 + e^{2x})^2} \\
 &= \frac{-4e^{2x}}{(1 + e^{2x})^2}.
 \end{aligned}$$

[5] (c) We use the Product Rule twice:

$$\begin{aligned}
 y' &= [x]' \tan(x) \sec(x) + x[\tan(x) \sec(x)]' \\
 &= 1 \cdot \tan(x) \sec(x) + x([\tan(x)]' \sec(x) + \tan(x)[\sec(x)]') \\
 &= \tan(x) \sec(x) + x(\sec^2(x) \cdot \sec(x) + \tan(x) \cdot \sec(x) \tan(x)) \\
 &= \tan(x) \sec(x) + x \sec^3(x) + x \tan^2(x) \sec(x).
 \end{aligned}$$

[5] (d) First we rewrite the function using the properties of logarithms:

$$\begin{aligned}
 y &= \ln(x^7 e^x) - \ln(\cot^2(x) \sqrt{x^4 + 1}) \\
 &= [\ln(x^7) + \ln(e^x)] - [\ln(\cot^2(x)) + \ln((x^4 + 1)^{\frac{1}{2}})] \\
 &= 7 \ln(x) + x - 2 \ln(\cot(x)) - \frac{1}{2} \ln(x^4 + 1) \\
 y' &= 7 \cdot \frac{1}{x} + 1 - 2 \cdot \frac{1}{\cot(x)} [\cot(x)]' - \frac{1}{2} \cdot \frac{1}{x^4 + 1} [x^4 + 1]' \\
 &= \frac{7}{x} + 1 - 2 \cdot \frac{1}{\cot(x)} \cdot [-\csc^2(x)] - \frac{1}{2} \cdot \frac{1}{x^4 + 1} \cdot 4x^3 \\
 &= \frac{7}{x} + 1 + \frac{2 \csc^2(x)}{\cot(x)} - \frac{2x^3}{x^4 + 1}.
 \end{aligned}$$

[7] 4. (a) We differentiate both sides of the equation with respect to x , using the Product Rule on

the righthand side:

$$\begin{aligned}
 \frac{d}{dx}[\sin(x) + 3 \cos(y)] &= \frac{d}{dx}[x^2 y] \\
 \cos(x) + 3[-\sin(y)] \frac{dy}{dx} + \frac{d}{dx}[x^2]y + x^2 \frac{d}{dx}[y] & \\
 \cos(x) - 3 \sin(y) \frac{dy}{dx} &= 2xy + x^2 \frac{dy}{dx} \\
 \cos(x) - 2xy &= x^2 \frac{dy}{dx} + 3 \sin(y) \frac{dy}{dx} \\
 \cos(x) - 2xy &= [x^2 + 3 \sin(y)] \frac{dy}{dx} \\
 \frac{dy}{dx} &= \frac{\cos(x) - 2xy}{x^2 + 3 \sin(y)}.
 \end{aligned}$$

[2] (b) The slope of the tangent line is

$$m = \frac{\cos(0) - 2 \cdot 0 \cdot \frac{\pi}{2}}{0^2 + 3 \sin\left(\frac{\pi}{2}\right)} = \frac{1 - 0}{0 + 3} = \frac{1}{3}.$$

Hence the equation of the tangent line is

$$y - \frac{\pi}{2} = \frac{1}{3}(x - 0) \quad \implies \quad y = \frac{1}{3}x + \frac{\pi}{2}.$$