MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Assignment 2

MATHEMATICS 1000

FALL 2025

SOLUTIONS

[4] 1. (a) This is a quasirational function for which direct substitution gives rise to a $\frac{0}{0}$ indeterminate form. We use the Rationalisation Method. Note that there is a radical in *both* the numerator and the denominator, so let's first try rationalising the numerator:

$$\lim_{x \to 4} \frac{3 - \sqrt{x+5}}{\sqrt{4x-7} - 3} = \lim_{x \to 4} \frac{3 - \sqrt{x+5}}{\sqrt{4x-7} - 3} \cdot \frac{3 + \sqrt{x+5}}{3 + \sqrt{x+5}}$$

$$= \lim_{x \to 4} \frac{9 - (x+5)}{\left(\sqrt{4x-7} - 3\right)\left(3 + \sqrt{x+5}\right)}$$

$$= \lim_{x \to 4} \frac{4 - x}{\left(\sqrt{4x-7} - 3\right)\left(3 + \sqrt{x+5}\right)}.$$

Now we'll rationalise the denominator:

$$\lim_{x \to 4} \frac{3 - \sqrt{x+5}}{\sqrt{4x-7} - 3} = \lim_{x \to 4} \frac{4 - x}{\left(\sqrt{4x-7} - 3\right) \left(3 + \sqrt{x+5}\right)} \cdot \frac{\sqrt{4x-7} + 3}{\sqrt{4x-7} + 3}$$

$$= \lim_{x \to 4} \frac{(4 - x) \left(\sqrt{4x-7} + 3\right)}{\left[(4x-7) - 9\right] \left(3 + \sqrt{x+5}\right)}$$

$$= \lim_{x \to 4} \frac{(4 - x) \left(\sqrt{4x-7} + 3\right)}{(4x-16) \left(3 + \sqrt{x+5}\right)}$$

$$= \lim_{x \to 4} \frac{-(x-4) \left(\sqrt{4x-7} + 3\right)}{4(x-4) \left(3 + \sqrt{x+5}\right)}$$

$$= \lim_{x \to 4} \frac{-\left(\sqrt{4x-7} + 3\right)}{4\left(3 + \sqrt{x+5}\right)}$$

$$= \frac{-6}{4(6)}$$

$$= -\frac{1}{4}.$$

[4] (b) Direct substitution results in an $\infty - \infty$ indeterminate form, so we need to rewrite the given function in a more useful way. By writing both terms over a common denominator, we have

$$\frac{3}{h} - \frac{6}{h\sqrt{7h+4}} = \frac{3\sqrt{7h+4}}{h\sqrt{7h+4}} - \frac{6}{h\sqrt{7h+4}} = \frac{3\sqrt{7h+4}-6}{h\sqrt{7h+4}}.$$

Now direct substitution results in a $\frac{0}{0}$ form, and since this is a quasirational function, we can use the Rationalisation Method:

$$\lim_{h \to 0} \left(\frac{3}{h} - \frac{6}{h\sqrt{7h+4}} \right) = \lim_{h \to 0} \frac{3\sqrt{7h+4} - 6}{h\sqrt{7h+4}} \cdot \frac{3\sqrt{7h+4} + 6}{3\sqrt{7h+4} + 6}$$

$$= \lim_{h \to 0} \frac{9(7h+4) - 36}{h\sqrt{7h+4} \left(3\sqrt{7h+4} + 6 \right)}$$

$$= \lim_{h \to 0} \frac{63h}{h\sqrt{7h+4} \left(3\sqrt{7h+4} + 6 \right)}$$

$$= \lim_{h \to 0} \frac{63}{\sqrt{7h+4} \left(3\sqrt{7h+4} + 6 \right)}$$

$$= \frac{63}{12(2)}$$

$$= \frac{21}{8}.$$

[3] (c) Direct substitution creates a $\frac{0}{0}$ form, so we need to make use of the special trigonometric limit

$$\lim_{t \to 0} \frac{\sin(t)}{t} = 1.$$

Let's concentrate on the sine function in the numerator first. There are three factors of $\sin(2x)$, so for each of these, we need a factor of 2x in the denominator in order to use the special limit. Since $(2x)^3 = 8x^3$, and there is already a factor of x^2 in the denominator, this means that we need to introduce a factor of 8x. Then we have

$$\lim_{x \to 0} \frac{\sin^3(2x)}{x^2 \sin(5x)} = \lim_{x \to 0} \frac{\sin^3(2x)}{x^2 \sin(5x)} \cdot \frac{8x}{8x}$$

$$= \lim_{x \to 0} \frac{\sin^3(2x)}{8x^3} \cdot \frac{8x}{\sin(5x)}$$

$$= \left(\lim_{x \to 0} \frac{\sin(2x)}{2x}\right)^3 \cdot \lim_{x \to 0} \frac{8x}{\sin(5x)}$$

$$= 1^3 \cdot \lim_{x \to 0} \frac{8x}{\sin(5x)}$$

$$= \lim_{x \to 0} \frac{8x}{\sin(5x)}$$

$$= \lim_{x \to 0} \frac{8x}{\sin(5x)}.$$

For the remaining sine function, we need 5x in the numerator to use the special limit:

$$\lim_{x \to 0} \frac{\sin^3(2x)}{x^2 \sin(5x)} = \lim_{x \to 0} \frac{8x}{\sin(5x)} \cdot \frac{5}{5}$$

$$= \frac{8}{5} \lim_{x \to 0} \frac{5x}{\sin(5x)}$$

$$= \frac{8}{5} \cdot 1$$

$$= \frac{8}{5}.$$

[3] 2. Since this is a piecewise function whose behaviour changes at x = -2, we must check the one-sided limits:

$$\lim_{x \to -2^{-}} f(x) = \lim_{x \to -2^{-}} k^{2}x = -2k^{2}$$

and

$$\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} (4k - x) = 4k + 2.$$

If the limit exists, then these one-sided limits must be equal, so we set

$$-2k^{2} = 4k + 2$$
$$2k^{2} + 4k + 2 = 0$$
$$2(k+1)^{2} = 0,$$

and hence k = -1.

(Note that the second part of the definition of f(x) did not affect our workings because it applies only for x = -2, which has no effect on the limit as x approaches -2.)

[6] 3. First we set

$$x^{3} + 3x^{2} = 0$$
$$x^{2}(x+3) = 0.$$

so we must check x = 0 and x = -3.

At x = 0, the numerator is 9, so x = 0 is a vertical asymptote.

At x = -3, the numerator is 0, so we take the limit:

$$\lim_{x \to -3} \frac{9 - x^2}{x^3 + 3x^2} = \lim_{x \to -3} \frac{-(x - 3)(x + 3)}{x^2(x + 3)} = \lim_{x \to -3} \frac{-(x - 3)}{x^2} = \frac{6}{9} = \frac{2}{3}.$$

Since the limit exists, x = -3 is not a vertical asymptote.

Note that x^2 is always non-negative, so the sign of the denominator is entirely determined by the sign of x + 3. As $x \to 0^-$, then, the denominator becomes a small positive number so, bearing in mind that the numerator is positive,

$$\lim_{x \to 0^-} f(x) = \infty.$$

As $x \to 0^+$, the denominator again becomes a small positive number, so

$$\lim_{x \to 0^+} f(x) = \infty$$

as well.