MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Section 2.4 **Math 1000 Worksheet** Fall 2023

SOLUTIONS

1. (a) $$\frac{dy}{dx} = \frac{d}{dx}[x^3-1](x^4+3x^2-x) + \frac{d}{dx}[x^4+3x^2-x](x^3-1)$$
$$= (3x^2-0)(x^4+3x^2-x) + (4x^3+6x-1)(x^3-1)$$
$$= 7x^6+15x^4-8x^3-6x+1$$

(b) $$f'(x) = (x+1)'(2x+1)(3x+1) + [(2x+1)(3x+1)]'(x+1)$$
$$= (1+0)(2x+1)(3x+1) + [(2x+1)'(3x+1)+(3x+1)'(2x+1)](x+1)$$
$$= (2x+1)(3x+1) + [(2+0)(3x+1)+(3+0)(2x+1)](x+1)$$
$$= 18x^2+22x+6$$

(c) $$g'(x) = \frac{(ax^2+b)'(cx^2-d)-(cx^2-d)'(ax^2+b)}{(cx^2-d)^2}$$
$$= \frac{2ax(cx^2-d)-2cx(ax^2+b)}{(cx^2-d)^2} = \frac{-2(ad+bc)x}{(cx^2-d)^2}$$

(d) Rewrite: $f(x) = \dfrac{1}{\frac{x^3+4x^2+4}{x+4}} = \dfrac{x+4}{x^3+4x^2+4}$

Differentiate: $$f'(x) = \frac{(x+4)'(x^3+4x^2+4)-(x^3+4x^2+4)'(x+4)}{(x^3+4x^2+4)^2}$$
$$= \frac{(x^3+4x^2+4)-(3x^2+8x)(x+4)}{(x^3+4x^2+4)^2}$$
$$= \frac{-2x^3-16x^2-32x+4}{(x^3+4x^2+4)^2}$$

2. Let $A(x) = \dfrac{f(x)}{g(x)}$ so

$$\frac{d}{dx}[A(x)] = \lim_{h\to 0}\frac{A(x+h)-A(x)}{h} = \lim_{h\to 0}\frac{\frac{f(x+h)}{g(x+h)}-\frac{f(x)}{g(x)}}{h}$$
$$= \lim_{h\to 0}\frac{f(x+h)g(x)-f(x)g(x+h)}{hg(x)g(x+h)}$$
$$= \lim_{h\to 0}\frac{1}{g(x)g(x+h)}\cdot\lim_{h\to 0}\frac{f(x+h)g(x)-f(x)g(x+h)}{h}$$
$$= \frac{1}{[g(x)]^2}\cdot\lim_{h\to 0}\frac{f(x+h)g(x)-f(x)g(x+h)}{h}.$$

Much like the proof of the Product Rule, we now subtract and add $f(x)g(x)$ to the numerator in the limit:

$$\begin{aligned}
\frac{d}{dx}[A(x)] &= \frac{1}{[g(x)]^2} \lim_{h\to 0} \frac{f(x+h)g(x) - f(x)g(x) + f(x)g(x) - f(x)g(x+h)}{h} \\
&= \frac{1}{[g(x)]^2} \cdot \left[\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \cdot g(x) - \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} \cdot f(x) \right] \\
&= \frac{\frac{d}{dx}[f(x)]g(x) - \frac{d}{dx}[g(x)]f(x)}{[g(x)]^2}.
\end{aligned}$$