MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 2.3 | **Math 1000 Worksheet** | FALL 2023

SOLUTIONS

1. (a) $\dfrac{dy}{dx} = 2\left(\dfrac{3}{4}x^{-\frac{1}{4}}\right) - 0 = \dfrac{3}{2x^{\frac{1}{4}}}$

   (b) Rewrite: $f(x) = 32x^5$

   Differentiate: $f'(x) = 32(5x^4) = 160x^4$

   (c) $V'(r) = \dfrac{4}{3}\pi(3r^2) = 4\pi r^2$

   (d) Rewrite: $y = 4x^{\frac{1}{2}} - 2x^{-3} - x$

   Differentiate: $\dfrac{dy}{dx} = 4\left(\dfrac{1}{2}x^{-\frac{1}{2}}\right) - 2(-3x^{-4}) - 1 = \dfrac{2}{\sqrt{x}} + \dfrac{6}{x^4} - 1$

   (e) Rewrite: $f(t) = 3t^4 + 24t^3 - t^2 - 8t$

   Differentiate: $f'(t) = 3(4t^3) + 24(3t^2) - 2t - 8 = 12t^3 + 72t^2 - 2t - 8$

   (f) Rewrite: $g(x) = \dfrac{5}{2} - \dfrac{1}{2}x^{-1}$

   Differentiate: $g'(x) = 0 - \dfrac{1}{2}(-x^{-2}) = \dfrac{1}{2x^2}$

2. When the pebble strikes the ground, $s(t) = 0$ so we solve:

$$-4.9t^2 - 14.7t + 343 = -4.9(t^2 + 3t - 70) = -4.9(t+10)(t-7) = 0,$$

giving $t = -10$ or $t = 7$. However, we assume that the pebble was dropped at $t = 0$ so we can ignore the negative answer; hence the pebble must strike the ground at 7 sec. The velocity function is

$$v(t) = s'(t) = -9.8t - 14.7$$

so we compute $v(7) = -9.8(7) - 14.7 = -83.3$. Hence the pebble is travelling at a velocity of -83.3 m/sec.

3. (a) We are told that $s(0) = 0$, and since $s(0) = D$, we immediately have $D = 0$ and therefore

$$s(t) = At^3 + Bt^2 + Ct.$$

   We also know that $v(0) = 0$, and

$$v(t) = s'(t) = A(3t^2) + B(2t) + C(1) = 3At^2 + 2Bt + C.$$

Thus $v(0) = C$ and so $C = 0,$ which means that

$$s(t) = At^3 + Bt^2.$$

Finally, we are given that $s(2) = s(6) = -36$. This tells us that

$$8A + 4B = -36 \quad \text{and} \quad 216A + 36B = -36.$$

Solving this system of equations gives $A = 2$ and $B = -13.$

(b) We now have

$$s(t) = 2t^3 - 13t^2 \quad \text{and} \quad v(t) = s'(t) = 6t^2 - 26t.$$

We want to solve the equation $v(t) = 0$, so we set

$$6t^2 - 26t = 2t(3t - 13) = 0,$$

and therefore either $t = 0$ or $t = \frac{13}{3} \approx 4.33$. Hence the object is again at rest at approximately 4.33 seconds. Its position at this time is

$$s\left(\frac{13}{3}\right) = 2\left(\frac{13}{3}\right)^3 - 13\left(\frac{13}{3}\right)^2 = -\frac{2197}{27} \approx -81.37 \text{ metres.}$$