MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

TEST 1 | **MATHEMATICS 1000-004** | FALL 2025

SOLUTIONS

[12] 1. (a) $f(-3) = 0$

(b) $\lim\limits_{x \to -3^-} f(x) = -4$

(c) $\lim\limits_{x \to -3^+} f(x) = -4$

(d) $\lim\limits_{x \to -3} f(x) = -4$

(e) $f(2) = 3$

(f) $\lim\limits_{x \to 2^-} f(x) = -2$

(g) $\lim\limits_{x \to 2^+} f(x) = 3$

(h) $\lim\limits_{x \to 2} f(x)$ does not exist (because the one-sided limits are not equal)

(i) $f(0)$ is undefined

(j) $\lim\limits_{x \to 0^-} f(x) = \infty$

(k) $\lim\limits_{x \to 0^+} f(x) = -\infty$

(l) $\lim\limits_{x \to 0} f(x)$ does not exist (and we cannot assign ∞ or $-\infty$ because the one-sided limits do not agree)

[6] 2. (a) Direct substitution results in a $\frac{0}{0}$ indeterminate form. Since this is a quasirational function, we use the Rationalisation Method:

$$\begin{aligned}
\lim_{x \to -1} \frac{3 - \sqrt{5 - 4x}}{1 - x^2} &= \lim_{x \to -1} \frac{3 - \sqrt{5 - 4x}}{1 - x^2} \cdot \frac{3 + \sqrt{5 - 4x}}{3 + \sqrt{5 - 4x}} \\
&= \lim_{x \to -1} \frac{9 - (5 - 4x)}{(1 - x^2)\left(3 + \sqrt{5 - 4x}\right)} \\
&= \lim_{x \to -1} \frac{4x + 4}{(1 - x^2)\left(3 + \sqrt{5 - 4x}\right)} \\
&= \lim_{x \to -1} \frac{4(x + 1)}{-(x - 1)(x + 1)\left(3 + \sqrt{5 - 4x}\right)} \\
&= \lim_{x \to -1} \frac{4}{-(x - 1)\left(3 + \sqrt{5 - 4x}\right)} \\
&= \frac{4}{-(-2)(3 + 3)} \\
&= \frac{1}{3}.
\end{aligned}$$

[5] (b) Direct substitution yields a $\frac{0}{0}$ indeterminate form. First we write the given function as a rational function, and then use the Cancellation Method:

$$\begin{aligned}
\lim_{x\to 4}\frac{x(x+4)^{-1}-2x^{-1}}{x-4} &= \lim_{x\to 4}\frac{\frac{x}{x+4}-\frac{2}{x}}{x-4}\\
&= \lim_{x\to 4}\left(\frac{x^2}{x(x+4)}-\frac{2(x+4)}{x(x+4)}\right)\cdot\frac{1}{x-4}\\
&= \lim_{x\to 4}\frac{x^2-2x-8}{x(x+4)(x-4)}\\
&= \lim_{x\to 4}\frac{(x-4)(x+2)}{x(x+4)(x-4)}\\
&= \lim_{x\to 4}\frac{x+2}{x(x+4)}\\
&= \frac{6}{4\cdot 8}\\
&= \frac{3}{16}.
\end{aligned}$$

[5] (c) We consider the one-sided limits. Since $|x| = x$ for $x \geq 0$, we have

$$\lim_{x\to 0^+}\frac{5x-|x|}{|x|+4x} = \lim_{x\to 0^+}\frac{5x-x}{x+4x} = \lim_{x\to 0^+}\frac{4x}{5x} = \lim_{x\to 0^+}\frac{4}{5} = \frac{4}{5}.$$

Since $|x| = -x$ for $x < 0$, we have

$$\lim_{x\to 0^-}\frac{5x-|x|}{|x|+4x} = \lim_{x\to 0^-}\frac{5x-(-x)}{-x+4x} = \lim_{x\to 0^-}\frac{6x}{3x} = \lim_{x\to 0^-} 2 = 2.$$

Thus $\lim_{x\to 0}\frac{5x-|x|}{|x|+4x}$ does not exist because the one-sided limits disagree.

[3] 3. (a) Since $f(x)$ is a rational function, we need only evaluate one of the limits at infinity:

$$\begin{aligned}
\lim_{x\to\infty} f(x) &= \lim_{x\to\infty}\frac{2x^3-4x^2+5x}{8x(9-x^2)}\\
&= \lim_{x\to\infty}\frac{2x^3-4x^2+5x}{72x-8x^3}\cdot\frac{\frac{1}{x^3}}{\frac{1}{x^3}}\\
&= \lim_{x\to\infty}\frac{2-\frac{4}{x}+\frac{5}{x^2}}{\frac{72}{x^2}-8}\\
&= \frac{2-0+0}{0-8}\\
&= -\frac{1}{4}.
\end{aligned}$$

Thus the line $y = -\dfrac{1}{4}$ is the only horizontal asymptote.

[9] (b) We set

$$8x(9 - x^2) = 0 \quad \Longrightarrow \quad -8x(x-3)(x+3) = 0$$

so $x = 0$, $x = 3$ or $x = -3$. These are all points of discontinuity (since they make the function undefined) and possible vertical asymptotes.
For $x = 0$, we see that $f(0) = \frac{0}{0}$ so we must take the limit:

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{2x^3 - 4x^2 + 5x}{8x(9-x^2)} = \lim_{x\to 0} \frac{x(2x^2 - 4x + 5)}{8x(9-x^2)} = \lim_{x\to 0} \frac{2x^2 - 4x + 5}{8(9-x^2)} = \frac{5}{72}.$$

Since the limit exists, $x = 0$ is a removable discontinuity and hence cannot be a vertical asymptote.
For $x = 3$, we see that $f(3) = \frac{33}{0}$. Since this is a $\frac{K}{0}$ form, $\lim\limits_{x\to 3} f(x)$ does not exist and so $x = 3$ is a non-removable discontinuity and a vertical asymptote.
For $x = -3$, we see that $f(-3) = \frac{-105}{0}$. Since this is also a $\frac{K}{0}$ form, $\lim\limits_{x\to -3} f(x)$ does not exist and so $x = -3$ is a non-removable discontinuity and a vertical asymptote as well.