

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

TEST 1

MATHEMATICS 1000-003

FALL 2025

SOLUTIONS

- [12] 1. (a) $f(0)$ is undefined
- (b) $\lim_{x \rightarrow 0^-} f(x) = \infty$
- (c) $\lim_{x \rightarrow 0^+} f(x) = -\infty$
- (d) $\lim_{x \rightarrow 0} f(x)$ does not exist (and we cannot assign ∞ or $-\infty$ because the one-sided limits do not agree)
- (e) $f(-3) = 0$
- (f) $\lim_{x \rightarrow -3^-} f(x) = -4$
- (g) $\lim_{x \rightarrow -3^+} f(x) = -4$
- (h) $\lim_{x \rightarrow -3} f(x) = -4$
- (i) $f(2) = 3$
- (j) $\lim_{x \rightarrow 2^-} f(x) = -2$
- (k) $\lim_{x \rightarrow 2^+} f(x) = 3$
- (l) $\lim_{x \rightarrow 2} f(x)$ does not exist (because the one-sided limits are not equal)
- [5] 2. (a) Direct substitution yields a $\frac{0}{0}$ indeterminate form. First we write the given function as a rational function, and then use the Cancellation Method:

$$\begin{aligned}
 \lim_{x \rightarrow 4} \frac{x(x+4)^{-1} - 2x^{-1}}{x-4} &= \lim_{x \rightarrow 4} \frac{\frac{x}{x+4} - \frac{2}{x}}{x-4} \\
 &= \lim_{x \rightarrow 4} \left(\frac{x^2}{x(x+4)} - \frac{2(x+4)}{x(x+4)} \right) \cdot \frac{1}{x-4} \\
 &= \lim_{x \rightarrow 4} \frac{x^2 - 2x - 8}{x(x+4)(x-4)} \\
 &= \lim_{x \rightarrow 4} \frac{(x-4)(x+2)}{x(x+4)(x-4)} \\
 &= \lim_{x \rightarrow 4} \frac{x+2}{x(x+4)} \\
 &= \frac{6}{4 \cdot 8} \\
 &= \frac{3}{16}.
 \end{aligned}$$

- [6] (b) Direct substitution results in a $\frac{0}{0}$ indeterminate form. Since this is a quasirational function, we use the Rationalisation Method:

$$\begin{aligned}
 \lim_{x \rightarrow -1} \frac{3 - \sqrt{5 - 4x}}{1 - x^2} &= \lim_{x \rightarrow -1} \frac{3 - \sqrt{5 - 4x}}{1 - x^2} \cdot \frac{3 + \sqrt{5 - 4x}}{3 + \sqrt{5 - 4x}} \\
 &= \lim_{x \rightarrow -1} \frac{9 - (5 - 4x)}{(1 - x^2)(3 + \sqrt{5 - 4x})} \\
 &= \lim_{x \rightarrow -1} \frac{4x + 4}{(1 - x^2)(3 + \sqrt{5 - 4x})} \\
 &= \lim_{x \rightarrow -1} \frac{4(x + 1)}{-(x - 1)(x + 1)(3 + \sqrt{5 - 4x})} \\
 &= \lim_{x \rightarrow -1} \frac{4}{-(x - 1)(3 + \sqrt{5 - 4x})} \\
 &= \frac{4}{-(-2)(3 + 3)} \\
 &= \frac{1}{3}.
 \end{aligned}$$

- [5] (c) We consider the one-sided limits. Since $|x| = x$ for $x \geq 0$, we have

$$\lim_{x \rightarrow 0^+} \frac{5x - |x|}{|x| + 4x} = \lim_{x \rightarrow 0^+} \frac{5x - x}{x + 4x} = \lim_{x \rightarrow 0^+} \frac{4x}{5x} = \lim_{x \rightarrow 0^+} \frac{4}{5} = \frac{4}{5}.$$

Since $|x| = -x$ for $x < 0$, we have

$$\lim_{x \rightarrow 0^-} \frac{5x - |x|}{|x| + 4x} = \lim_{x \rightarrow 0^-} \frac{5x - (-x)}{-x + 4x} = \lim_{x \rightarrow 0^-} \frac{6x}{3x} = \lim_{x \rightarrow 0^-} 2 = 2.$$

Thus $\lim_{x \rightarrow 0} \frac{5x - |x|}{|x| + 4x}$ does not exist because the one-sided limits disagree.

- [3] 3. (a) Since $f(x)$ is a rational function, we need only evaluate one of the limits at infinity:

$$\begin{aligned}
 \lim_{x \rightarrow \infty} f(x) &= \lim_{x \rightarrow \infty} \frac{6x^3 - 7x^2 + 3x}{9x(4 - x^2)} \\
 &= \lim_{x \rightarrow \infty} \frac{6x^3 - 7x^2 + 3x}{36x - 9x^3} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} \\
 &= \lim_{x \rightarrow \infty} \frac{6 - \frac{7}{x} + \frac{3}{x^2}}{\frac{36}{x^2} - 9} \\
 &= \frac{6 - 0 + 0}{0 - 9} \\
 &= -\frac{2}{3}.
 \end{aligned}$$

Thus the line $y = -\frac{2}{3}$ is the only horizontal asymptote.

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(b) We set

$$9x(4 - x^2) = 0 \implies -9x(x - 2)(x + 2) = 0$$

so $x = 0$, $x = 2$ or $x = -2$. These are all points of discontinuity (since they make the function undefined) and possible vertical asymptotes.

For $x = 0$, we see that $f(0) = \frac{0}{0}$ so we must take the limit:

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{6x^3 - 7x^2 + 3x}{9x(4 - x^2)} = \lim_{x \rightarrow 0} \frac{x(6x^2 - 7x + 3)}{9x(4 - x^2)} = \lim_{x \rightarrow 0} \frac{6x^2 - 7x + 3}{9(4 - x^2)} = \frac{3}{36} = \frac{1}{12}.$$

Since the limit exists, $x = 0$ is a removable discontinuity and hence cannot be a vertical asymptote.

For $x = 2$, we see that $f(2) = \frac{26}{0}$. Since this is a $\frac{K}{0}$ form, $\lim_{x \rightarrow 2} f(x)$ does not exist and so $x = 2$ is a non-removable discontinuity and a vertical asymptote.

For $x = -2$, we see that $f(-2) = \frac{-82}{0}$. Since this is also a $\frac{K}{0}$ form, $\lim_{x \rightarrow -2} f(x)$ does not exist and so $x = -2$ is a non-removable discontinuity and a vertical asymptote as well.