MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 1.2 **Math 1000 Worksheet** FALL 2023

SOLUTIONS

1. (a) $f(0)$ $= 3$
   (b) $\lim_{x\to 0^-} f(x)$ $= 0$
   (c) $\lim_{x\to 0^+} f(x)$ $= 3$
   (d) $\lim_{x\to 0} f(x)$ does not exist because the one-sided limits are not equal
   (e) $f(3)$ $= -1$
   (f) $\lim_{x\to 3^-} f(x)$ $= 3$
   (g) $\lim_{x\to 3^+} f(x)$ $= 3$
   (h) $\lim_{x\to 3} f(x)$ $= 3$
   (i) $f(4)$ $= 0$
   (j) $\lim_{x\to 4} f(x)$ $= 0$
   (k) $f(-2)$ is undefined because $x = -2$ is a vertical asymptote
   (ℓ) $\lim_{x\to -2^-} f(x)$ $= -\infty$
   (m) $\lim_{x\to -2^+} f(x)$ $= \infty$
   (n) $\lim_{x\to -2} f(x)$ does not exist because the one-sided limits are not equal

2. Note that $|9x| = 9x$ for $x > 0$ and $|9x| = -9x$ for $x < 0$. Thus, for $x > 0$,

$$g(x) = \frac{7x - 9x}{4x} = \frac{-2x}{4x} = -\frac{1}{2},$$

while for $x < 0$,

$$g(x) = \frac{7x - (-9x)}{4x} = \frac{16x}{4x} = 4.$$

Finally, for $x = 0$, we have division by zero, so $g(0)$ is undefined. Hence we can write

$$g(x) = \begin{cases} 4 & \text{for } x < 0 \\ -\frac{1}{2} & \text{for } x > 0 \end{cases}$$

with the graph is given in Figure 1.

Now we have:

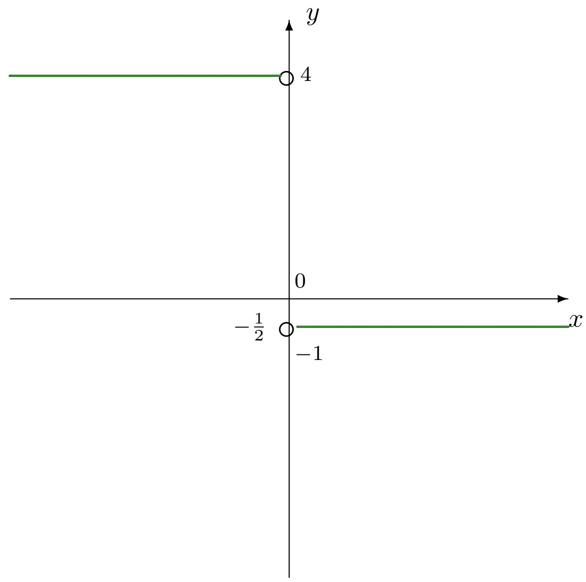

Figure 1: The graph of $f(x) = \dfrac{7x - |9x|}{4x}$ for Section 1.2, Question 3.

(a) $\lim_{x\to 0-} f(x) = 4$

(b) $\lim_{x\to 0+} f(x) = -\dfrac{1}{2}$

(c) $\lim_{x\to 0} f(x)$ does not exist because the one sided-limits are not equal

(d) $\lim_{x\to 4} f(x) = -\dfrac{1}{2}$

(e) $\lim_{x\to -\frac{6}{5}} f(x) = 4$

3. (a) First we consider values to the left of $x = 4$:

x	3.5	3.9	3.99	3.999	3.9999
$f(x)$	0.94118	0.90722	0.90070	0.90007	0.90001

and then values to the right of $x = 4$:

x	4.5	4.1	4.01	4.001	4.0001
$f(x)$	0.86957	0.89320	0.89930	0.89993	0.89999

In both cases, it appears that the function is tending towards a value of 0.9 as x approaches 4. Hence we may conclude that

$$\lim_{x\to 4} \frac{2x^2 - 7x - 4}{3x^2 - 14x + 8} = 0.9 = \frac{9}{10}.$$

(b) First we consider values to the left of $x = 0$:

x	-1	-0.5	-0.1	-0.01	-0.001
$f(x)$	-3.3860	-0.1657	-0.0822	-0.0800	-0.0800

and then values to the right of $x = 0$:

x	1	0.5	0.1	0.01	0.001
$f(x)$	-3.3860	-0.1657	-0.0822	-0.0800	-0.0800

Since the behaviour of the function is the same on either side of $x = 0$, we can conclude that

$$\lim_{x\to 0} \frac{\tan^2(x)}{\cos(5x) - 1} = -0.08 = -\frac{2}{25}.$$

(c) First we consider values to the left of $x = -1$:

x	-1.5	-1.1	-1.01	-1.001	-1.0001	-1.00001
$f(x)$	-7.333	-4.238	-3.795	-3.755	-3.7505	-3.7500

and then values to the right of $x = -1$:

x	-0.5	-0.9	-0.99	-0.999	-0.9999	-0.999999
$f(x)$	-2.16	-3.333	-3.705	-3.746	-3.7496	-3.74998

In each case, it seems that as $x \to -1$, the function is tending towards a value of -3.75 or $-\frac{15}{4}$. We can deduce that

$$\lim_{x\to -1} \frac{3x^2 - 9x - 12}{x^3 + 7x^2 + 15x + 9} = -\frac{15}{4}.$$

(d) First we consider values to the left of $x = -3$:

x	-3.5	-3.1	-3.01	-3.001
$f(x)$	-90	-2130	-210300	-21003000

and then values to the right of $x = -3$:

x	-2.5	-2.9	-2.99	-2.999
$f(x)$	-78	-2070	-209700	-20997000

In each case, it seems that as $x \to -3$, the function is becoming an unboundedly large negative number. Thus the limit does not exist, but we can write

$$\lim_{x\to -3} \frac{3x^2 - 9x - 12}{x^3 + 7x^2 + 15x + 9} = -\infty.$$