MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

TEST 2 **MATHEMATICS 1000-003** October 26th, 2022

Name	**MUN Number**

1. For each of the following, indicate your answer by checking the corresponding box. (You may check only <u>one</u> box per question. If more than one box is checked, you will not receive any credit for that question.)

[3] (a) Which, if any, of the following <u>is</u> a property of the horizontal asymptotes to a function $f(x)$?

- ☐ a horizontal asymptote describes the behaviour of $f(x)$ as x becomes unboundedly large (positively or negatively)
- ☐ a horizontal asymptote describes the manner in which $f(x)$ becomes unboundedly large (positively or negatively) as x approaches a real number p
- ☐ the graph of $f(x)$ may never cross a horizontal asymptote
- ☐ $f(x)$ may have any number of horizontal asymptotes
- ☐ none of the above is a property of horizontal asymptotes

[3] (b) Which, if any, of the following statements would ensure that a function $f(x)$ <u>has</u> a removable discontinuity at a point $x = p$?

- ☐ $f(p)$ is defined, but $\lim_{x \to p} f(x)$ does not exist
- ☐ $\lim_{x \to p} f(x)$ exists, but $f(p)$ is undefined
- ☐ the definition of $f(x)$ changes at $x = p$
- ☐ factors of $(x - p)$ can be cancelled from both the numerator and denominator of $f(x)$
- ☐ none of the above would ensure this

[3] (c) Which, if any, of the following situations is <u>impossible</u> for a function $f(x)$ at a point $x = p$?

- ☐ $f(x)$ is differentiable at $x = p$ but not continuous at $x = p$
- ☐ $f(x)$ is continuous at $x = p$ but not differentiable at $x = p$
- ☐ $f(x)$ is both continuous and differentiable at $x = p$
- ☐ $f(x)$ is neither continuous nor differentiable at $x = p$
- ☐ none of the above is impossible

[3] (d) Which, if any, of the following does <u>not</u> provide graphical evidence that a function $f(x)$ is non-differentiable at a point $x = p$?

- ☐ $f(x)$ has an abrupt change or "sharp corner" at $x = p$
- ☐ $f(x)$ has a vertical asymptote at $x = p$
- ☐ $f(x)$ has a vertical tangent line at $x = p$
- ☐ $f(x)$ has a horizontal tangent line at $x = p$
- ☐ none of the above provides this evidence

[3] (e) Which of the following is <u>neither</u> an example of, nor equivalent to, a rate of change?

- ☐ the slope of a tangent line to a curve
- ☐ the derivative of a function
- ☐ the infection rate of a virus
- ☐ the velocity of an object moving in a straight line
- ☐ all of the above are examples of, or are equivalent to, a rate of change

[5] 2. Identify, with justification, any horizontal asymptotes to the graph of $f(x) = \dfrac{2x^3(6x-1)}{(3x^2+4)^2}$.

[8] 3. Given the function

$$f(x) = \begin{cases} x^2 + 5kx, & \text{for } x < 2 \\ 3k^2 - 4, & \text{for } x = 2 \\ k^2x + 4x + 4, & \text{for } x > 2 \end{cases}$$

use the definition of continuity to determine all values of the constant k for which $f(x)$ is continuous at $x = 2$.

4. Consider the function $f(x) = \dfrac{x}{2x+5}$.

[8] (a) Use the limit definition of the derivative to find $f'(x)$.

[4] (b) Find the equation of the line that is tangent to the curve $y = \dfrac{x}{2x+5}$ at $x = -3$.

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[40]