MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 6 | **MATHEMATICS 1000** | FALL 2022

SOLUTIONS

[3] 1. (a) We use the Chain Rule, followed by the Product Rule:

$$\begin{aligned} f'(t) &= \cos(t^3e^t) \cdot [t^3e^t]' \\ &= \cos(t^3e^t) \cdot [(t^3)'e^t + (e^t)'t^3] \\ &= \cos(t^3e^t) \cdot [3t^2e^t + t^3e^t] \\ &= t^2(t+3)e^t\cos(t^3e^t). \end{aligned}$$

[4] (b) Because this function is in the form $[f(x)]^{g(x)}$, we must use logarithmic differentiation:

$$\begin{aligned} \ln(y) &= \ln\left([\sin(x)]^{\cos(x)}\right) \\ &= \cos(x)\ln(\sin(x)) \\ \frac{d}{dx}[\ln(y)] &= \frac{d}{dx}[\cos(x)\ln(\sin(x))] \\ \frac{1}{y}\cdot\frac{dy}{dx} &= -\sin(x)\ln(\sin(x)) + \frac{1}{\sin(x)}\cdot\frac{d}{dx}[\sin(x)]\cdot\cos(x) \\ &= -\sin(x)\ln(\sin(x)) + \frac{1}{\sin(x)}\cdot\cos^2(x) \\ &= -\sin(x)\ln(\sin(x)) + \cos(x)\cot(x) \\ \frac{dy}{dx} &= y[\cos(x)\cot(x) - \sin(x)\ln(\sin(x))] \\ &= [\sin(x)]^{\cos(x)}[\cos(x)\cot(x) - \sin(x)\ln(\sin(x))]. \end{aligned}$$

[3] (c) First we simplify the function:

$$\begin{aligned} f(x) &= \ln(x\tan^7(3x)) - \ln(\sqrt[3]{x^2+1}) \\ &= \ln(x) + \ln(\tan^7(3x)) - \ln(\sqrt[3]{x^2+1}) \\ &= \ln(x) + 7\ln(\tan(3x)) - \frac{1}{3}\ln(x^2+1). \end{aligned}$$

Now we differentiate:

$$\begin{aligned} f'(x) &= \frac{1}{x} + 7\cdot\frac{1}{\tan(3x)}\cdot[\tan(3x)]' - \frac{1}{3}\cdot\frac{1}{x^4+1}\cdot[x^2+1]' \\ &= \frac{1}{x} + 7\cdot\frac{1}{\tan(x)}\cdot 3\sec^2(3x) - \frac{1}{3}\cdot\frac{1}{x^2+1}\cdot 2x \\ &= \frac{1}{x} + \frac{21\sec^2(3x)}{\tan(3x)} - \frac{2x}{3(x^2+1)}. \end{aligned}$$

[5] 2. Differentiating both sides of the equation with respect to x, we have

$$\begin{aligned}
\frac{d}{dx}[x^4 + 8y^3] &= \frac{d}{dx}[8x^2y] \\
4x^3 + 8(3y^2)\frac{dy}{dx} &= 8(2x)y + 8x^2\frac{dy}{dx} \\
24y^2\frac{dy}{dx} - 8x^2\frac{dy}{dx} &= 16xy - 4x^3 \\
\frac{dy}{dx}(24y^2 - 8x^2) &= 16xy - 4x^3 \\
\frac{dy}{dx} &= \frac{16xy - 4x^3}{24y^2 - 8x^2} \\
&= \frac{4xy - x^3}{6y^2 - 2x^2}.
\end{aligned}$$

Thus the slope of the tangent line at the point $(3, \frac{3}{2})$ is

$$\frac{dy}{dx} = \frac{4(3)\left(\frac{3}{2}\right) - 3^3}{6\left(\frac{3}{2}\right)^2 - 2(3^2)} = \frac{-9}{-\frac{9}{2}} = 2.$$

The equation of the tangent line, then, is

$$y - \frac{3}{2} = 2(x - 3) \quad \Longrightarrow \quad \boxed{y = 2x - \frac{9}{2}.}$$

[5] 3. We have

$$\begin{aligned}
\ln(y) &= \ln\left(\frac{(2x+7)^3e^x}{x^9\sin^2(x)}\right) \\
&= \ln((2x+7)^3e^x) - \ln(x^9\sin^2(x)) \\
&= \ln((2x+7)^3) + \ln(e^x) - \ln(x^9) - \ln(\sin^2(x)) \\
&= 3\ln(2x+7) + x - 9\ln(x) - 2\ln(\sin(x)).
\end{aligned}$$

Now we can differentiate both sides with respect to x and obtain

$$\begin{aligned}
[\ln(y)]' &= [3\ln(2x+7) + x - 9\ln(x) - 2\ln(\sin(x))]' \\
\frac{1}{y} \cdot y' &= 3 \cdot \frac{1}{2x+7} \cdot [2x+7]' + 1 - 9 \cdot \frac{1}{x} - 2 \cdot \frac{1}{\sin(x)} \cdot [\sin(x)]' \\
&= 3 \cdot \frac{1}{2x+7} \cdot 2 + 1 - \frac{9}{x} - 2 \cdot \frac{1}{\sin(x)} \cdot \cos(x) \\
&= \frac{6}{2x+7} + 1 - \frac{9}{x} - 2\cot(x) \\
y' &= y\left[\frac{6}{2x+7} + 1 - \frac{9}{x} - 2\cot(x)\right] \\
&= \frac{(2x+7)^3 e^x}{x^9 \sin^2(x)} \left[\frac{6}{2x+7} + 1 - \frac{9}{x} - 2\cot(x)\right].
\end{aligned}$$