MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Section 4.6	**Math 1000 Worksheet**	Fall 2022

SOLUTIONS

1. (a) This is a $\frac{0}{0}$ indeterminate form:

$$\lim_{x\to 0} \frac{6^x - 2^x}{x} \overset{\text{H}}{=} \lim_{x\to 0} \frac{6^x \ln(6) - 2^x \ln(2)}{1} = \ln(6) - \ln(2) = \ln(3).$$

(b) This is a $\frac{0}{0}$ indeterminate form:

$$\lim_{x\to 0^+} \frac{1 - \cos\left(\sqrt{x}\right)}{x} \overset{\text{H}}{=} \lim_{x\to 0^+} \frac{\sin\left(\sqrt{x}\right) \cdot \frac{1}{2\sqrt{x}}}{1} = \lim_{x\to 0^+} \frac{\sin\left(\sqrt{x}\right)}{2\sqrt{x}}$$

$$\overset{\text{H}}{=} \lim_{x\to 0^+} \frac{\cos\left(\sqrt{x}\right) \cdot \frac{1}{2\sqrt{x}}}{\frac{1}{\sqrt{x}}} = \lim_{x\to 0^+} \frac{\cos\left(\sqrt{x}\right)}{2} = \frac{1}{2}.$$

(c) This is a $\frac{0}{0}$ indeterminate form:

$$\lim_{x\to 0} \frac{\sin(mx)}{\sin(nx)} \overset{\text{H}}{=} \lim_{x\to 0} \frac{m\cos(mx)}{n\cos(nx)} = \frac{m \cdot 1}{n \cdot 1} = \frac{m}{n}.$$

(d) This is an $\frac{\infty}{\infty}$ indeterminate form:

$$\lim_{x\to \infty} \frac{\ln(1 + e^{2x})}{x} \overset{\text{H}}{=} \lim_{x\to \infty} \frac{\frac{1}{1+e^{2x}} \cdot 2e^{2x}}{1} = \lim_{x\to \infty} \frac{2e^{2x}}{1 + e^{2x}} \overset{\text{H}}{=} \lim_{x\to \infty} \frac{4e^{2x}}{2e^{2x}} = \lim_{x\to \infty} 2 = 2.$$

(e) This is an $\frac{\infty}{\infty}$ indeterminate form:

$$\begin{aligned}
\lim_{x\to \infty} \frac{[\ln(x)]^3}{x^2} &\overset{\text{H}}{=} \lim_{x\to \infty} \frac{3[\ln(x)]^2 \cdot \frac{1}{x}}{2x} \\
&= \lim_{x\to \infty} \frac{3[\ln(x)]^2}{2x^2} \\
&\overset{\text{H}}{=} \lim_{x\to \infty} \frac{6\ln(x) \cdot \frac{1}{x}}{4x} \\
&= \lim_{x\to \infty} \frac{3\ln(x)}{2x^2} \\
&\overset{\text{H}}{=} \lim_{x\to \infty} \frac{3 \cdot \frac{1}{x}}{4x} \\
&= \lim_{x\to \infty} \frac{3}{4x^2} \\
&= 0.
\end{aligned}$$

(f) This is an $\frac{\infty}{\infty}$ indeterminate form:

$$\lim_{x\to\infty} \frac{x^2+1}{x\ln(x)} \overset{\text{H}}{=} \lim_{x\to\infty} \frac{2x}{x\left(\frac{1}{x}\right)+\ln(x)} = \lim_{x\to\infty} \frac{2x}{1+\ln(x)} \overset{\text{H}}{=} \lim_{x\to\infty} \frac{2}{\frac{1}{x}} = \lim_{x\to\infty} 2x = \infty.$$

(g) This is an $\infty \cdot 0$ indeterminate form:

$$\lim_{x\to\frac{\pi}{2}^-} \sec(7x)\cos(3x) = \lim_{x\to\frac{\pi}{2}^-} \frac{\cos(3x)}{\cos(7x)} \overset{\text{H}}{=} \lim_{x\to\frac{\pi}{2}^-} \frac{-3\sin(3x)}{-7\sin(7x)} = \frac{-3(-1)}{-7(-1)} = \frac{3}{7}.$$

(h) This is an $\infty - \infty$ indeterminate form:

$$\begin{aligned}\lim_{x\to 1}\left(\frac{1}{\ln(x)} - \frac{1}{x-1}\right) &= \lim_{x\to 1} \frac{x-1-\ln(x)}{(x-1)\ln(x)} \\ &\overset{\text{H}}{=} \lim_{x\to 1} \frac{1-\frac{1}{x}}{(x-1)\cdot\frac{1}{x}+\ln(x)} \\ &= \lim_{x\to 1} \frac{x-1}{x-1+x\ln(x)} \\ &\overset{\text{H}}{=} \lim_{x\to 1} \frac{1}{1+\ln(x)+x\cdot\frac{1}{x}} \\ &= \lim_{x\to 1} \frac{1}{2+\ln(x)} \\ &= \frac{1}{2}.\end{aligned}$$

(i) This is a 0^0 indeterminate form. Let $y = \sin(x)^{\tan(x)}$ so $\ln(y) = \tan(x)\ln(\sin(x))$. Then

$$\begin{aligned}\lim_{x\to 0^+} \tan(x)\ln(\sin(x)) &= \lim_{x\to 0^+} \frac{\ln(\sin(x))}{\cot(x)} \overset{\text{H}}{=} \lim_{x\to 0^+} \frac{\frac{1}{\sin(x)}[\cos(x)]}{-\csc^2(x)} \\ &= \lim_{x\to 0^+} [-\sin(x)\cos(x)] = 0.\end{aligned}$$

Thus $\lim_{x\to 0^+} (\sin(x))^{\tan(x)} = e^0 = 1.$

(j) This is an ∞^0 indeterminate form. Let $y = (x+e^x)^{\frac{1}{x}}$ so $\ln(y) = \frac{1}{x}\cdot\ln(x+e^x)$. Then

$$\begin{aligned}\lim_{x\to\infty} \frac{1}{x}\cdot\ln(x+e^x) &= \lim_{x\to\infty} \frac{\ln(x+e^x)}{x} \overset{\text{H}}{=} \lim_{x\to\infty} \frac{\frac{1}{x+e^x}\cdot(1+e^x)}{1} = \lim_{x\to\infty} \frac{1+e^x}{x+e^x} \\ &\overset{\text{H}}{=} \lim_{x\to\infty} \frac{e^x}{1+e^x} \overset{\text{H}}{=} \lim_{x\to\infty} \frac{e^x}{e^x} = \lim_{x\to\infty} 1 = 1.\end{aligned}$$

Thus $\lim_{x\to\infty} (x+e^x)^{\frac{1}{x}} = e^1 = e.$

(k) This is a 1^∞ indeterminate form. Let $y = \cos(3x))^{\frac{5}{x}}$ so $\ln(y) = \frac{5}{x}\ln(\cos(3x))$. Then

$$\begin{aligned}\lim_{x\to 0}\frac{5}{x}\ln(\cos(3x)) = \lim_{x\to 0}\frac{5\ln(\cos(3x))}{x} &\overset{\text{H}}{=} \lim_{x\to 0}\frac{5\cdot\frac{1}{\cos(3x)}\cdot[-3\sin(3x)]}{1} \\ &= \lim_{x\to 0}[-15\tan(3x)] = 0.\end{aligned}$$

Thus $\lim_{x\to 0}(\cos(3x))^{\frac{5}{x}} = e^0 = 1.$

(ℓ) This is a 1^∞ indeterminate form. Let $y = \left(1+\frac{a}{x}\right)^{bx}$ so $\ln(y) = bx\ln\left(1+\frac{a}{x}\right)$. Then

$$\lim_{x\to\infty} bx\ln\left(1+\frac{a}{x}\right) = \lim_{x\to\infty}\frac{b\ln\left(1+\frac{a}{x}\right)}{\frac{1}{x}} \overset{\text{H}}{=} \lim_{x\to\infty}\frac{\frac{b}{1+\frac{a}{x}}\cdot\left(-\frac{a}{x^2}\right)}{-\frac{1}{x^2}} = \lim_{x\to\infty}\frac{ab}{1+\frac{a}{x}} = ab.$$

Thus $\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^{bx} = e^{ab}.$