

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 3.7

Math 1000 Worksheet

FALL 2022

SOLUTIONS

1. We compute each derivative in turn:

$$\begin{aligned}f'(x) &= (x^2)'e^x + (e^x)'x^2 = 2xe^x + x^2e^x \\f''(x) &= [(2x)'e^x + (e^x)'(2x)] + [(x^2)'e^x + (e^x)'x^2] \\&= [2e^x + 2xe^x] + [2xe^x + x^2e^x] = 2e^x + 4xe^x + x^2e^x \\f'''(x) &= 2(e^x)' + [(4x)'e^x + (e^x)'(4x)] + [(x^2)'e^x + (e^x)'x^2] \\&= 2e^x + [4e^x + 4xe^x] + [2xe^x + x^2e^x] \\&= 6e^x + 6xe^x + x^2e^x.\end{aligned}$$

2. We have

$$\begin{aligned}f'(x) &= (x)' \sin(x) + [\sin(x)]'x = (1) \sin(x) + \cos(x)x = \sin(x) + x \cos(x) \\f''(x) &= [\sin(x)]' + (x)' \cos(x) + [\cos(x)]'x = \cos(x) + (1) \cos(x) + [-\sin(x)]x \\&= 2 \cos(x) - x \sin(x) \\f'''(x) &= [2 \cos(x)]' - (x)' \sin(x) - [\sin(x)]'x = -2 \sin(x) - (1) \sin(x) - \cos(x)x \\&= -3 \sin(x) - x \cos(x) \\f^{(4)}(x) &= [-3 \sin(x)]' - (x)' \cos(x) - x[\cos(x)]' \\&= -3 \cos(x) - (1) \cos(x) - x[-\sin(x)] = -4 \cos(x) + x \sin(x).\end{aligned}$$

3. We use the Chain Rule to obtain the first derivative:

$$y' = \sec^2(x^2) \cdot (x^2)' = 2x \sec^2(x^2).$$

Next we need to use both the Chain Rule and the Product Rule to find the second derivative:

$$\begin{aligned}y'' &= (2x)' \sec^2(x^2) + [\sec^2(x^2)]'(2x) \\&= 2 \sec^2(x^2) + 2 \sec(x^2) \cdot [\sec(x^2)]'(2x) \\&= 2 \sec^2(x^2) + 2 \sec(x^2) \cdot \sec(x^2) \tan(x^2) \cdot (x^2)'(2x) \\&= 2 \sec^2(x^2) + 2 \sec(x^2) \cdot \sec(x^2) \tan(x^2) \cdot (2x)(2x) \\&= 2 \sec^2(x^2) + 8x^2 \sec^2(x^2) \tan(x^2).\end{aligned}$$

4. We begin by finding the first derivative:

$$\begin{aligned}\frac{d}{dx}[y] &= \frac{d}{dx}[2x - y^2] \\ \frac{dy}{dx} &= 2 - 2y \frac{dy}{dx} \\ \frac{dy}{dx} + 2y \frac{dy}{dx} &= 2 \\ \frac{dy}{dx}(1 + 2y) &= 2 \\ \frac{dy}{dx} &= \frac{2}{1 + 2y}.\end{aligned}$$

We differentiate again, substituting the expression for $\frac{dy}{dx}$ where possible:

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left[\frac{2}{1 + 2y} \right] = \frac{0 - 2 \cdot 2 \frac{dy}{dx}}{(1 + 2y)^2} = \frac{-4 \frac{dy}{dx}}{(1 + 2y)^2} = \frac{-4 \left(\frac{2}{1 + 2y} \right)}{(1 + 2y)^2} = \frac{-8}{(1 + 2y)^3}.$$

5. We differentiate implicitly to find the first derivative:

$$\begin{aligned}\frac{d}{dx} [\sqrt{x} + \sqrt{y}] &= \frac{d}{dx} [2] \\ \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}y^{-\frac{1}{2}} \frac{dy}{dx} &= 0 \\ \frac{dy}{dx} &= - \left(\frac{y}{x} \right)^{\frac{1}{2}}.\end{aligned}$$

Differentiating implicitly a second time gives us

$$\begin{aligned}\frac{d^2y}{dx^2} &= -\frac{1}{2} \left(\frac{y}{x} \right)^{-\frac{1}{2}} \left(\frac{\frac{d}{dx}[y]x - \frac{d}{dx}[x]y}{x^2} \right) = -\frac{1}{2} \left(\frac{y}{x} \right)^{-\frac{1}{2}} \left(\frac{\left(\frac{dy}{dx} \right) x - (1)y}{x^2} \right) \\ &= -\frac{1}{2} \left(\frac{x}{y} \right)^{\frac{1}{2}} \left(\frac{- \left(\frac{y}{x} \right)^{\frac{1}{2}} x - y}{x^2} \right) = \frac{\sqrt{x} + \sqrt{y}}{2x^{\frac{3}{2}}}.\end{aligned}$$

But remember that $\sqrt{x} + \sqrt{y} = 2$, so this becomes

$$\frac{d^2y}{dx^2} = \frac{2}{2x^{\frac{3}{2}}} = x^{-\frac{3}{2}}.$$

6. We have

$$\begin{aligned}v(t) = s'(t) &= \frac{(49t - 10)'(t + 10) - (t + 10)'(49t - 10)}{(t + 10)^2} \\&= \frac{(49)(t + 10) - (1)(49t - 10)}{(t + 10)^2} = \frac{500}{(t + 10)^2} = \frac{500}{t^2 + 20t + 100} \\a(t) = s''(t) &= \frac{(500)'(t^2 + 20t + 100) - (t^2 + 20t + 100)'(500)}{(t^2 + 20t + 100)^2} \\&= \frac{0 - (2t + 20)(500)}{(t^2 + 20t + 100)^2} = \frac{-1000(t + 10)}{(t + 10)^4} = -\frac{1000}{(t + 10)^3}.\end{aligned}$$

Therefore,

$$v(0) = \frac{500}{100} = 5 \quad \text{and} \quad a(0) = \frac{-1000}{1000} = -1.$$

Hence the puppy's initial velocity is **5 cm/sec** and its initial acceleration is **-1 cm/sec²**.