MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 3.5 **Math 1000 Worksheet** FALL 2022

## SOLUTIONS

1. (a) We want an angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose sine is $-\frac{\sqrt{2}}{2}$, so

$$\arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}.$$

(b) We want an angle between 0 and $\pi$ whose cosine is $-\frac{\sqrt{2}}{2}$, so

$$\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}.$$

(c) First note that

$$\operatorname{arcsec}\left(\frac{2\sqrt{3}}{3}\right) = \arccos\left(\frac{3}{2\sqrt{3}}\right) = \arccos\left(\frac{\sqrt{3}}{2}\right),$$

so we want an angle between 0 and $\pi$ whose cosine is $\frac{\sqrt{3}}{2}$. Hence

$$\operatorname{arcsec}(\sqrt{2}) = \frac{\pi}{6}.$$

(d) Note that $\frac{9\pi}{4} > \frac{\pi}{2}$ so we cannot simply use the cancellation equation. But $\tan\left(\frac{9\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) = 1$ and so

$$\arctan\left(\tan\left(\frac{9\pi}{4}\right)\right) = \frac{\pi}{4}.$$

(e) Let $\theta = \arccos\left(\frac{5}{13}\right)$. Then we can construct a right triangle with an interior angle $\theta$, adjacent sidelength 5 and hypotenuse of length 13. By the Pythagorean theorem, the remaining side has length

$$\sqrt{13^2 - 5^2} = \sqrt{144} = 12 \quad \Longrightarrow \quad \sin\left(\arccos\left(\frac{5}{13}\right)\right) = \sin(\theta) = \frac{12}{13}.$$

(f) Let $\theta = \arctan(2)$. We construct a right triangle with interior angle $\theta$, opposite sidelength 2 and adjacent sidelength 1. Then the length of the hypotenuse is

$$\sqrt{1^2 + 2^2} = \sqrt{5} \quad \Longrightarrow \quad \cos(\arctan(2)) = \cos(\theta) = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}.$$

2. (a) $$y' = \frac{1}{\ln(x)\sqrt{[\ln(x)]^2 - 1}} \cdot [\ln(x)]' = \frac{1}{x\ln(x)\sqrt{[\ln(x)]^2 - 1}}$$

(b) $$y' = (x^2)' \arctan(3x) + [\arctan(3x)]' x^2$$
$$= 2x\arctan(3x) + \frac{1}{1+9x^2} \cdot (3x)' \cdot x^2 = 2x\arctan(3x) + \frac{3x^2}{1+9x^2}$$

(c) $$y' = \frac{1}{\sqrt{1-\tan^2(t^2)}} \cdot [\tan(t^2)]'$$
$$= \frac{1}{\sqrt{1-\tan^2(t^2)}} \cdot \sec^2(t^2) \cdot (t^2)' = \frac{2t\sec^2(t^2)}{\sqrt{1-\tan^2(t^2)}}$$

(d) First we have

$$y' = \sec^2(\arcsin(t^2)) \cdot [\arcsin(t^2)]' = \sec^2(\arcsin(t^2)) \cdot \frac{1}{\sqrt{1-(t^2)^2}} \cdot (t^2)'$$
$$= \frac{2t\sec^2(\arcsin(t^2))}{\sqrt{1-t^4}}.$$

But note that if $\theta = \arcsin(t^2)$ then we can construct a right triangle with $t^2$ as the length of the side opposite $\theta$, and 1 as the length of the hypotenuse. The length of the adjacent side must be

$$\sqrt{1^2 - (t^2)^2} = \sqrt{1-t^4}$$

so

$$\sec(\arcsin(t^2)) = \sec(\theta) = \frac{1}{\sqrt{1-t^4}} \implies \sec^2(\arcsin(t^2)) = \frac{1}{1-t^4}.$$

Thus

$$y' = \frac{2t \cdot \frac{1}{1-t^4}}{\sqrt{1-t^4}} = \frac{2t}{(1-t^4)^{\frac{3}{2}}}.$$

3. First, observe that

$$f'(x) = \frac{1}{\sqrt{1-\left(\frac{x-2}{2}\right)^2}} \cdot \frac{1}{2} - \frac{2}{\sqrt{1-\frac{x}{4}}} \cdot \frac{1}{4\sqrt{x}} = \frac{1}{2\sqrt{x-\frac{1}{4}x^2}} - \frac{1}{2\sqrt{x-\frac{1}{4}x^2}} = 0.$$

Thus the tangent line must be horizontal. When $x = 2$,

$$y = f(2) = \arcsin(0) - 2\arcsin\left(\frac{\sqrt{2}}{2}\right) = 0 - 2\left(\frac{\pi}{4}\right) = -\frac{\pi}{2}.$$

Hence the equation of the tangent line is $y = -\frac{\pi}{2}$.

4. Differentiating implicitly, we have

$$\begin{aligned}
\frac{d}{dx}\left[\sqrt{1-x^2y^2}\right] &= \frac{d}{dx}[\arccos(xy)] \\
\frac{1}{2\sqrt{1-x^2y^2}} \cdot \frac{d}{dx}[1-x^2y^2] &= -\frac{1}{\sqrt{1-(xy)^2}} \cdot \frac{d}{dx}[xy] \\
\frac{1}{2\sqrt{1-x^2y^2}}\left(-2xy^2 - 2x^2y\frac{dy}{dx}\right) &= -\frac{1}{\sqrt{1-x^2y^2}}\left(y + x\frac{dy}{dx}\right) \\
\left(-xy^2 - x^2y\frac{dy}{dx}\right) &= -y - x\frac{dy}{dx} \\
\frac{dy}{dx}(-x^2y + x) &= xy^2 - y \\
\frac{dy}{dx} &= \frac{xy^2 - y}{-x^2y + x} = \frac{y(xy-1)}{-x(xy-1)} = -\frac{y}{x}.
\end{aligned}$$

5. Let $y = \arccos(x)$ so $\cos(y) = x$. Differentiating implicitly, we have

$$\begin{aligned}
\frac{d}{dx}[\cos(y)] &= \frac{d}{dx}[x] \\
-\sin(y)\frac{dy}{dx} &= 1 \\
\frac{dy}{dx} &= -\frac{1}{\sin(y)}.
\end{aligned}$$

Since $\sin^2(y) + \cos^2(y) = 1$, we know that $\sin(y) = \pm\sqrt{1-\cos^2(y)}$. However, for $0 \leq y \leq \pi$, $\sin(y) \geq 0$, and so $\sin(y) = \sqrt{1-\cos^2(y)} = \sqrt{1-x^2}$. Hence

$$\frac{dy}{dx} = \frac{d}{dx}[\arccos(x)] = -\frac{1}{\sqrt{1-x^2}}.$$