

MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 3.4

Math 1000 Worksheet

FALL 2024

SOLUTIONS

1. (a) $f'(x) = 2^{\ln(x)} \ln(2) \cdot [\ln(x)]' = \frac{2^{\ln(x)} \ln(2)}{x}$
- (b) Rewrite: $y = \log_3(7) + 5 \log_3(x)$
Differentiate: $\frac{dy}{dx} = 0 + 5 \cdot \frac{1}{x \ln(3)} = \frac{5}{x \ln(3)}$
- (c) Rewrite: $y = \ln(5) - 7 \ln(x)$
Differentiate: $\frac{dy}{dx} = 0 - 7 \cdot \frac{1}{x} = -\frac{7}{x}$
- (d) Rewrite: $g(x) = \ln(x^2 - 6) - \ln(x^2 + 4)$
Differentiate: $g'(x) = \frac{1}{x^2 - 6} \cdot (x^2 - 6)' - \frac{1}{x^2 + 4} \cdot (x^2 + 4)'$
 $= \frac{1}{x^2 - 6} \cdot (2x) - \frac{1}{x^2 + 4} \cdot (2x) = \frac{2x}{x^2 - 6} - \frac{2x}{x^2 + 4}$
 $= \frac{2x(x^2 + 4) - 2x(x^2 - 6)}{(x^2 - 6)(x^2 + 4)} = \frac{20x}{(x^2 - 6)(x^2 + 4)}$

2. (a) We take logarithms of both sides and then differentiate:

$$\begin{aligned}\ln(y) &= \ln \left[\frac{x^{3x}(2-x)^4}{(5x^5+1)\sqrt[3]{\csc^2(x)}} \right] \\ &= 3x \ln(x) + 4 \ln(2-x) - \ln(5x^5+1) - \frac{2}{3} \ln(\csc(x)) \\ \frac{1}{y} \frac{dy}{dx} &= \frac{d}{dx} [3x] \ln(x) + \frac{d}{dx} [\ln(x)] (3x) + 4 \cdot \frac{1}{2-x} \cdot \frac{d}{dx} [2-x] \\ &\quad - \frac{1}{5x^5+1} \cdot \frac{d}{dx} [5x^5+1] - \frac{2}{3} \cdot \frac{1}{\csc(x)} \cdot \frac{d}{dx} [\csc(x)] \\ &= 3 \ln(x) + \frac{1}{x} \cdot (3x) + \frac{4}{2-x} \cdot (-1) - \frac{1}{5x^5+1} \cdot (25x^4) \\ &\quad - \frac{2}{3 \csc(x)} \cdot [-\csc(x) \cot(x)] \\ &= 3 \ln(x) + 3 - \frac{4}{2-x} - \frac{25x^4}{5x^5+1} + \frac{2}{3} \cot(x) \\ \frac{dy}{dx} &= \frac{x^{3x}(2-x)^4}{(5x^5+1)\sqrt[3]{\csc^2(x)}} \left[3 \ln(x) + 3 - \frac{4}{2-x} - \frac{25x^4}{5x^5+1} + \frac{2}{3} \cot(x) \right].\end{aligned}$$

(b) We first take logarithms of both sides:

$$\begin{aligned}\ln(y) &= \ln \left[\frac{\sqrt{x^2 + 1}}{(3x + 2)[\tan(x)]^x} \right] \\ &= \frac{1}{2} \ln(x^2 + 1) - \ln(3x + 2) - x \ln(\tan(x)).\end{aligned}$$

Then we differentiate:

$$\begin{aligned}\frac{1}{y} \frac{dy}{dx} &= \frac{1}{2} \cdot \frac{1}{x^2 + 1} \cdot \frac{d}{dx}[x^2 + 1] - \frac{1}{3x + 2} \cdot \frac{d}{dx}[3x + 2] - \frac{d}{dx}[x] \ln(\tan(x)) \\ &\quad - \frac{d}{dx}[\ln(\tan(x))]x \\ &= \frac{1}{2} \cdot \frac{1}{x^2 + 1} \cdot (2x) - \frac{1}{3x + 2} \cdot (3) - (1) \ln(\tan(x)) - \frac{1}{\tan(x)} \cdot \frac{d}{dx}[\tan(x)]x \\ &= \frac{x}{x^2 + 1} - \frac{3}{3x + 2} - \ln(\tan(x)) - x \csc(x) \sec(x) \\ \frac{dy}{dx} &= \frac{\sqrt{x^2 + 1}}{(3x + 2)[\tan(x)]^x} \left[\frac{x}{x^2 + 1} - \frac{3}{3x + 2} - \ln(\tan(x)) - x \csc(x) \sec(x) \right].\end{aligned}$$