

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 3.3

Math 1000 Worksheet

FALL 2021

SOLUTIONS

1. We differentiate both sides with respect to x , giving

$$\begin{aligned}\frac{d}{dx}[y^2] &= \frac{d}{dx}[x^4 - x^6] \\ 2y \frac{dy}{dx} &= 4x^3 - 6x^5 \\ \frac{dy}{dx} &= \frac{4x^3 - 6x^5}{2y} = \frac{2x^3 - 3x^5}{y}.\end{aligned}$$

2. We differentiate both sides with respect to x , giving

$$\begin{aligned}\frac{d}{dx}[2y^3 + 3y^2] &= \frac{d}{dx}[(x^2 - 1)^2] \\ 6y^2 \frac{dy}{dx} + 6y \frac{dy}{dx} &= 2(x^2 - 1) \cdot 2x \\ \frac{dy}{dx}(6y^2 + 6y) &= 4x(x^2 - 1) \\ \frac{dy}{dx} &= \frac{2x(x^2 - 1)}{3y(y + 1)}.\end{aligned}$$

3. We differentiate implicitly, using the Product Rule on the righthand side:

$$\begin{aligned}\frac{d}{dx}[9y^2] &= \frac{d}{dx}[(y - 1)^2(x^2 + y^2)] \\ 18y \frac{dy}{dx} &= 2(y - 1) \frac{dy}{dx} \cdot (x^2 + y^2) + \left(2x + 2y \frac{dy}{dx}\right) \cdot (y - 1)^2 \\ -2x(y - 1)^2 &= \frac{dy}{dx}[2(y - 1)(x^2 + y^2) + 2y(y - 1)^2 - 18y] \\ \frac{dy}{dx} &= \frac{-2x(y - 1)^2}{2(y - 1)(x^2 + y^2) + 2y(y - 1)^2 - 18y} \\ &= \frac{-x(y - 1)^2}{x^2y - x^2 + 2y^3 - 3y^2 - 8y}.\end{aligned}$$

4. We differentiate implicitly:

$$\begin{aligned}\frac{d}{dx} \left[\frac{xy}{\pi} \right] &= \frac{d}{dx} [\cos(x+y)] \\ \frac{1}{\pi} \left(\frac{d}{dx}[x]y + x \frac{d}{dx}[y] \right) &= -\sin(x+y) \cdot \frac{d}{dx}[x+y] \\ \frac{1}{\pi} \left(y + x \frac{dy}{dx} \right) &= -\sin(x+y) \left(1 + \frac{dy}{dx} \right) \\ \frac{dy}{dx} \left[\frac{x}{\pi} + \sin(x+y) \right] &= -\frac{y}{\pi} - \sin(x+y) \\ \frac{dy}{dx} &= \frac{-\frac{y}{\pi} - \sin(x+y)}{\frac{x}{\pi} + \sin(x+y)} = \frac{-y - \pi \sin(x+y)}{x + \pi \sin(x+y)}.\end{aligned}$$

At the point $(0, \frac{\pi}{2})$,

$$\frac{dy}{dx} = \frac{-\frac{\pi}{2} - \pi \sin\left(\frac{\pi}{2}\right)}{0 + \pi \sin\left(\frac{\pi}{2}\right)} = \frac{-\frac{3\pi}{2}}{\pi} = -\frac{3}{2}$$

so then the equation of the tangent line is

$$y - \frac{\pi}{2} = -\frac{3}{2}(x - 0) \quad \Longrightarrow \quad y = -\frac{3}{2}x + \frac{\pi}{2}.$$

The slope of the normal line is $\frac{2}{3}$, the negative reciprocal of the slope of the tangent line. Hence the equation of the normal line is

$$y - \frac{\pi}{2} = \frac{2}{3}(x - 0) \quad \Longrightarrow \quad y = \frac{2}{3}x + \frac{\pi}{2}.$$