

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 2.4

Math 1000 Worksheet

FALL 2024

SOLUTIONS

$$\begin{aligned}
 1. \text{ (a) } \frac{dy}{dx} &= \frac{d}{dx}[x^3 - 1](x^4 + 3x^2 - x) + \frac{d}{dx}[x^4 + 3x^2 - x](x^3 - 1) \\
 &= (3x^2 - 0)(x^4 + 3x^2 - x) + (4x^3 + 6x - 1)(x^3 - 1) \\
 &= 7x^6 + 15x^4 - 8x^3 - 6x + 1
 \end{aligned}$$

$$\begin{aligned}
 \text{(b) } f'(x) &= (x+1)'(2x+1)(3x+1) + [(2x+1)(3x+1)]'(x+1) \\
 &= (1+0)(2x+1)(3x+1) + [(2x+1)'(3x+1) + (3x+1)'(2x+1)](x+1) \\
 &= (2x+1)(3x+1) + [(2+0)(3x+1) + (3+0)(2x+1)](x+1) \\
 &= 18x^2 + 22x + 6
 \end{aligned}$$

$$\begin{aligned}
 \text{(c) } g'(x) &= \frac{(ax^2 + b)'(cx^2 - d) - (cx^2 - d)'(ax^2 + b)}{(cx^2 - d)^2} \\
 &= \frac{2ax(cx^2 - d) - 2cx(ax^2 + b)}{(cx^2 - d)^2} = \frac{-2(ad + bc)x}{(cx^2 - d)^2}
 \end{aligned}$$

$$\text{(d) Rewrite: } f(x) = \frac{1}{\frac{x^3 + 4x^2 + 4}{x+4}} = \frac{x+4}{x^3 + 4x^2 + 4}$$

$$\begin{aligned}
 \text{Differentiate: } f'(x) &= \frac{(x+4)'(x^3 + 4x^2 + 4) - (x^3 + 4x^2 + 4)'(x+4)}{(x^3 + 4x^2 + 4)^2} \\
 &= \frac{(x^3 + 4x^2 + 4) - (3x^2 + 8x)(x+4)}{(x^3 + 4x^2 + 4)^2} \\
 &= \frac{-2x^3 - 16x^2 - 32x + 4}{(x^3 + 4x^2 + 4)^2}
 \end{aligned}$$

$$2. \text{ Let } A(x) = \frac{f(x)}{g(x)} \text{ so}$$

$$\begin{aligned}
 \frac{d}{dx}[A(x)] &= \lim_{h \rightarrow 0} \frac{A(x+h) - A(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{f(x+h)g(x) - f(x)g(x+h)}{hg(x)g(x+h)} \\
 &= \lim_{h \rightarrow 0} \frac{1}{g(x)g(x+h)} \cdot \lim_{h \rightarrow 0} \frac{f(x+h)g(x) - f(x)g(x+h)}{h} \\
 &= \frac{1}{[g(x)]^2} \cdot \lim_{h \rightarrow 0} \frac{f(x+h)g(x) - f(x)g(x+h)}{h}.
 \end{aligned}$$

Much like the proof of the Product Rule, we now subtract and add $f(x)g(x)$ to the numerator in the limit:

$$\begin{aligned}\frac{d}{dx}[A(x)] &= \frac{1}{[g(x)]^2} \lim_{h \rightarrow 0} \frac{f(x+h)g(x) - f(x)g(x) + f(x)g(x) - f(x)g(x+h)}{h} \\ &= \frac{1}{[g(x)]^2} \cdot \left[\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \cdot g(x) - \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} \cdot f(x) \right] \\ &= \frac{\frac{d}{dx}[f(x)]g(x) - \frac{d}{dx}[g(x)]f(x)}{[g(x)]^2}.\end{aligned}$$