SOLUTIONS

[12]   1. (a) $f(-1) = 1$

(b) $\displaystyle\lim_{x \to -1^-} f(x) = -3$

(c) $\displaystyle\lim_{x \to -1^+} f(x) = 1$

(d) $\displaystyle\lim_{x \to -1} f(x)$ does not exist (because the one-sided limits do not agree)

(e) $f(2)$ is undefined

(f) $\displaystyle\lim_{x \to 2^-} f(x) = -2$

(g) $\displaystyle\lim_{x \to 2^+} f(x) = -2$

(h) $\displaystyle\lim_{x \to 2} f(x) = -2$

(i) $f(3)$ is undefined

(j) $\displaystyle\lim_{x \to 3^-} f(x) = \infty$

(k) $\displaystyle\lim_{x \to 3^+} f(x) = \infty$

(l) $\displaystyle\lim_{x \to 3} f(x) = \infty$

[4]   2. (a) First we consider values to the left of $x = -4$:

x	-5	-4.5	-4.1	-4.01	-4.001
$f(x)$	-0.6875	-0.8163	-0.9573	-0.9956	-0.9996

and then values to the right of $x = -4$:

x	-3	-3.5	-3.9	-3.99	-3.999
$f(x)$	-1.75	-1.280	-1.0464	-1.0045	-1.0004

We can deduce that

$$\lim_{x \to -4^-} f(x) = -1 \quad \text{and} \quad \lim_{x \to -4^+} f(x) = -1,$$

and since these agree, we can conclude that

$$\lim_{x \to -4} f(x) = -1.$$

[4]   (b) First we consider values to the left of $x = -1$:

x	-2	-1.5	-1.1	-1.01	-1.001
$f(x)$	-5	-16	-320	-30200	-3002000

and then values to the right of $x = -1$:

x	0	-0.5	-0.9	-0.99	-0.999
$f(x)$	-1	-8	-280	-29800	-2998000

We can deduce that

$$\lim_{x \to -1^-} f(x) = -\infty \quad \text{and} \quad \lim_{x \to -1^+} f(x) = -\infty.$$

Since the limits are infinite, $\lim_{x \to -1} f(x)$ does not exist, but we can write that

$$\lim_{x \to -1} f(x) = -\infty.$$