Contributed Talks

Trees within infinite primitive highly arc transitive digraphs

Daniela Amato
University of Oxford

The set of descendants of a vertex α in a digraph D is the set of vertices which are reachable from α by a directed path. We investigate the set of descendants of a vertex in an infinite primitive highly arc transitive digraph with finite out-valency m. Our results show that if m is equal to p, p^2 or p^3, where p is prime, then the set of descendants of a vertex is a (rooted) tree. Moreover, we conjecture that the same is true when m is equal to p^k for any $k \in \mathbb{N}$.

Partial linear spaces and their applications to SOMAs

John Arhin
Queen Mary, University of London

A *partial linear space* $S = (P, L)$ consists of a set P of *points* with a set L of *lines*, where each line is a subset of P (of cardinality greater than or equal to 2), such that every pair of points is contained in at most one line.

A *PLS*$(v,n;r)$ is a partial linear space where the set of points has size v, each line has size n and every point is contained in exactly r lines.

A *decomposition* of $S = (P, L)$, a PLS$(v,n;r)$, is a partition $\{L_1, \ldots, L_m\}$ of L, such that every point of P is contained in exactly r_i lines of L_i, for all $i = 1, \ldots, m$. We note that if there exists $L' \subseteq L$, where (P, L) is a PLS$(v,n;r)$, such that every point of P is contained in exactly r' lines of L', then it is a simple exercise to show that (P, L') is a PLS$(v,n;r')$.

Suppose that $\{L_1, \ldots, L_m\}$ is a decomposition of $S = (P, L)$, a PLS$(v,n;r)$, such that (P, L_i) is a PLS$(v,n;r_i)$, for all $i = 1, \ldots, m$. Then if $\{L_i\}$ is the only decomposition of the PLS$(v,n;r_i)$ (P, L_i), for all $i = 1, \ldots, m$, then $\{L_1, \ldots, L_m\}$ is said to be an *unrefinable decomposition* of S.

In this talk we will discuss the result that if $S = (P, L)$ is a PLS$(v,n;r)$ with $v < n^2$, then $\{L\}$ is an unique unrefinable decomposition of S. Next, we will discuss the result that every PLS$(n^2,n;r)$ has an unique unrefinable decomposition, and provide an efficient algorithm for its computation. This particular result implies that every SOMA (a generalistion of mutually orthogonal Latin squares) must have an unique unrefinable decomposition, which in turn answers a question of Soicher. We then look at some generalisations of these results, and we find that in some cases these results are best possible.

Error-correcting codes, permutation groups and covering designs

Robert Bailey
Queen Mary, University of London

We move away from the traditional setting for error-correcting codes, namely vector spaces over finite fields, and replace these with permutation groups. We draw upon sharply k-transitive groups and the general linear and affine general linear groups (among others) for examples, and describe a decoding algorithm which uses the complements of the blocks of a covering design.

Graphs with the Erdos-Ko-Rado Property

Peter Borg
Open University

Let G be a graph and let r be an integer with $r \geq 1$. We denote the family of independent r-sets of $V(G)$ by $I^{(r)}(G)$. Let $I_v^{(r)}(G) := \{A \in I^{(r)}(G) : v \in A\}$, and call such a family a *star*. An *anomalous* subfamily of $I^{(r)}G$ is an intersecting subfamily that is not a subfamily of any star. G is said to be (*strictly*) *r-EKR* if the largest star is (strictly) larger than any anomalous subfamily. The Erdos-Ko-Rado theorem states that if E_n is the empty graph of order n, then E_n is *r-EKR* for $n \geq 2r$ and strictly so for $n > 2r$. We will give a survey of other such results for certain classes of graphs, and we will discuss open problems and current research in this area.

Codes meeting the Grey-Rankin bound from quasi-symmetric designs

Carl Bracken
National University of Ireland, Maynooth

Given any $4n$ by $4n$ Hadamard matrix we can constuct a quasi-symmetric design with parameters $(32n^2 - 4n, 16n^2 - 4n, 16n^2 - 4n - 1)$, the incidence matrix of which yields a binary code with parameters $(32n^2 - 4n, 128n^2, 16n^2 - 4n)$. This code will meet the Grey-Rankin bound (an upper bound on the number of words in a self-complimentary code) with equality.

Can colouring hypergraphs help to prevent fires?

David Cariolaro
University of Reading

In firefighting it is extremely important to have full information about dangerous or hazardous materials (or combinations or those) in factories and private or public buildings well before the occurrence of a real emergency. Indeed, every factory, supermarket, school, etc. is required by law to fill a detailed list of all chemicals, explosives, flammable or toxic materials which are temporarily or permanently stored in the building, and a long list of prohibited substances is made available annually by the local Fire Brigade. Developing a systematic, precise and consistent notation to represent dangerous or potentially dangerous situations is, in fire prevention, a must. In this talk we shall attempt to support by real examples the claim that hypergraph colouring is the most natural way to study, mathematically, problems of this kind.

$L(2,1)$-labellings of graphs

Luis Cereceda
London School of Economics

Given a graph $G = (V,E)$, an $L(2,1)$ labelling of G is a function $l : V \to \mathbb{N}$ satisfying the following two conditions:

1. $|l(u) - l(v)| \geq 2$ for all $uv \in E$
2. $|l(u) - l(v)| \geq 1$ for all $u, v \in V$ with $d(u,v) = 2$

where $d(u,v)$ denotes the distance between vertices u and v. The difference between the largest and smallest labels (numbers) used in l is called the *span* of the labelling, and the minimum span over all labellings of G is called the *lambda number* of G, $\lambda(G)$.

The problem of computing $\lambda(G)$ for a given graph, which arises naturally in the context of frequency assignment, is in general NP-hard (as is to be expected, since $L(2,1)$ labellings generalise graph colouring). We examine exact results, bounds and algorithms for $\lambda(G)$ for specific classes of graphs, as well as some open problems.

Genetic Algorithms for search problems in Vershik groups

Matthew Craven
UMIST

Genetic algorithms were introduced by Holland in 1975 and after initial theoretical research, started to be used in applications from the early 1980s onwards, achieving some striking results. It is only recently that they have been applied to combinatorial group theory by Borovik and Myasnikov.

The theory of traces has many applications in mathematics and computer science, including concurrent systems and graph theory. They come from the combinatorial algebra of free and commutation monoids, where the use for algebraic means was introduced by Cartier and Foata. The latter monoid is where we apply solely commutator relations between some generators, but no other relations. We extend this to a group structure. We call these the trace groups, which are otherwise known as graph groups, right-angled Artin groups and free partially commutative groups.

We focus on Vershik groups, a subclass of trace groups, and examine the double coset problem in this setting. This problem has links in cryptography, which suggests possible attacks on group-theoretic problems across wider classes of groups.

Some results are presented, which suggests that the GA in the case of this problem may converge to a deterministic algorithm.

Whist tournaments

Leigh Ellison
University of Glasgow

Whist is a game involving four players split into two teams of two. A great deal of work has been done with regards to proving the existence of different kinds of tournaments of this type. This talk is going to focus on $\mathbb{Z}$-cyclic ordered/directed triplewhist tournaments, and some recent results involving them. Generalised whist tournaments will also be discussed in passing.

Distance and fractional isomorphism in Steiner triple systems

Tony Forbes
Open University
Joint work with M. J. Grannell and T. S. Griggs

For the purpose of this talk, a *configuration* is a finite set of triples of points where two triples intersect in at most one point. Two configurations $\mathcal{C}$ and $\mathcal{D}$ are *isomorphic*, $\mathcal{C} \cong \mathcal{D}$, if there is a permutation of the points, $\phi : \bigcup \mathcal{C} \to \bigcup \mathcal{D}$, which preserves triples; i.e. $T \in \mathcal{C}$ iff $\phi(T) \in \mathcal{D}$. A *Steiner triple system* of order v (STS(v)), is a pair $(V, \mathcal{B})$ where V is a set of points, $v = |V|$ and $\mathcal{B}$ is a configuration in which each pair of distinct elements of V occurs in precisely one triple.

Given two Steiner triple systems $S = (V,\mathcal{B})$ and $S' = (V,\mathcal{B}')$ on the same base set V, the distance between them is the smallest number of triples that need to be changed to transform S to a system isomorphic to S'. The natural object to study distance is a *trade*; that is, a pair of disjoint configurations $\{\mathcal{C},\mathcal{D}\}$, which cover the same pairs. Thus $|\mathcal{C}| = |\mathcal{D}|$ and a pair of points appears in a triple of $\mathcal{C}$ iff it is present in a triple of $\mathcal{D}$.

We present a complete 'distance table' for the 80 Steiner triple systems of order 15.

All trades where $|\mathcal{C}| \leq 8$ have isomorphic configurations, $\mathcal{C} \cong \mathcal{D}$, but this is not true in general. We are interested the effect on the STS(15) distance table if trades are restricted to those where $\mathcal{C} \cong \mathcal{D}$.

Closely related to trades is the concept of *fractional isomorphism*, introduced by Quattrocchi and Rinaldi in a paper of 1997. Two configurations $\mathcal{C}$ and $\mathcal{D}$ are said to be n^{-1}-*isomorphic* if there are partitions $\{\mathcal{C}_1,\mathcal{C}_2,\ldots,\mathcal{C}_n\}$ of $\mathcal{C}$ and $\{\mathcal{D}_1,\mathcal{D}_2,\ldots,\mathcal{D}_n\}$ of $\mathcal{D}$ such that $\mathcal{C}_i \cong \mathcal{D}_i$ for $i = 1,2,\ldots,n$. Two Steiner triple systems, $(V,\mathcal{B})$ and $(V',\mathcal{B}')$, are n^{-1}-isomorphic if $\mathcal{B}$ and $\mathcal{B}'$ are n^{-1}-isomorphic. We make use of the distance tables to investigate fractional isomorphism between pairs of Steiner triple systems of order 15.

An application of the FKG inequality to embeddings of rooted binary trees

Nicholas Georgiou
London School of Economics

If $\mathcal{F}$ is a finite distributive lattice and μ,α,β are non-negative functions on $\mathcal{F}$ such that $\mu(f)\mu(g) \leq \mu(f \vee g)\mu(f \wedge g)$ for all $f,g \in \mathcal{F}$, and α,β are both increasing (or both decreasing), then

$$\sum_{f\in\mathcal{F}} \mu(f)\alpha(f) \sum_{f\in\mathcal{F}} \mu(f)\beta(f) \leq \sum_{f\in\mathcal{F}} \mu(f) \sum_{f\in\mathcal{F}} \mu(f)\alpha(f)\beta(f).$$

This powerful result is known as the FKG inequality and it has been used to obtain many results of a probabilistic nature.

Let T^n be the complete binary tree of height n, with root 1_n as the maximum element. Let T be an arbitrary rooted binary tree. An embedding of T into T^n is a map ϕ from T to T^n such that $x > y$ in T if and only if $\phi(x) > \phi(y)$ in T^n.

We show that it is possible to place a distributive lattice structure on the set of all embeddings of T into T^n. Using the FKG inequality, we show that certain events on the lattice are positively correlated. We also show how this approach gives a simpler proof of a related theorem of Kubicki, Lehel and Morayne on the number of embeddings that map the root of T to 1_n.

Stable sets of maximal size in Kneser-type graphs

Cheng Yeaw Ku
Queen Mary, University of London

We introduce a family of vertex-transitive graphs with specified subgroups of automorphisms which generalise Kneser graphs and Cayley graphs of permutations. We give some results on the characterisation of stable sets of maximal size in these graphs.

The total-chromatic number of Paley graphs of square order

Eleni Maistrelli
University of Essex

For a prime power $q \equiv 1 \pmod 4$, a Paley graph P_q is the graph with vertex set the finite field on q elements, $\mathbf{F}_q$, and an edge between two of its vertices if and only if their difference is a non-zero square in $\mathbf{F}_q$.

The total-chromatic number $\chi''(G)$ of a graph G is the least number of colours needed to colour the vertices and edges of G so that adjacent vertices, incident edges and edges and their incident vertices receive different colours.

In this talk we are going to briefly explore the total-chromatic number and present a 'nice' proof about it in the special case of Paley graphs of square order.

Colouring random geometric graphs

Tobias Müller
University of Oxford

A random geometric graph is obtained by taking its vertices to be a sample $\{X_1, \dots, X_n\}$ from some probability measure ν on $\mathbb{R}^d$ and putting $X_i \sim X_j$ for $i \neq j$ whenever $l(X_i - X_j) < r = r(n)$, where l is some norm on $\mathbb{R}^d$ and $r \to 0$ as $n \to \infty$. Such graphs have for instance been studied in connection with the spread of epidemics, wireless computer networks and the channel assignment problem in mobile telecommunications. In this talk we will investigate the asymptotic behaviour of the chromatic number of these graphs as $n \to \infty$. We will establish various almost sure convergence results for the chromatic number and we will find that (the asymptotic behaviour of) the chromatic number can be characterised in terms of the the maximum density ν_{max} of ν, the quantity nr^d (which is related to the average degree), and the packing density of the l-unit ball (that is $\{x \in \mathbb{R}^d : l(x) < 1\}$). It turns out that the chromatic number undergoes a "phase change" when the average degree is $\Theta(\ln(n))$.

Discrete analysis

Tomas Nilsson
Mid-Sweden University

A sequence of $n+1$ points corresponds to a unique polynomial in $\mathcal{P}_n$. We use difference tables to expose this relationship. Some applications of these tables will be outlined briefly, e.g. interpolation and a kind of 'discrete analysis'. Alternatively we could just find some nice identities.

Derangements in the affine general linear group

Pablo Spiga
Queen Mary, University of London

We will compute explicitly the number of derangements of the affine general linear group, in its natural action, using combinatorial properties of partitions of integers.

Countable homogeneous coloured partial orders

Susana Torrezão de Sousa
University of Leeds

We give an initial description of the countable homogeneous coloured partial orders, countable partial orders with a colour function for which any finite isomorphism extends to an automorphism of the whole structure. These results extend Schmerl's classification in the countable case and provide interesting examples towards a full classification. The classification is divided into two major cases, the first of which, the "interdense case", in which all colours occur interdensely, gives rise to somewhat similar structures to those in the monochromatic case. The other case is handled by use of a suitable equivalence relation that recognises interdense components of the previous kind. It is then necessary to determine in what ways they fit together. Fraisse's construction plays an important role in the systematic analysis of some of these cases, giving rise to structures with interesting properties.

Witness sets for vectors

Ben Veal
London School of Economics

A witness set for a vector $x \in V$ is a set of coordinates that distinguish x from all other vectors in V. These sets are studied in learning theory where they represent a set of questions that may be used to distinguish a concept from other concepts in their class. I will present some results on witness sets of minimal cardinality for arbitrary length vectors, and would be interested to know of any applications/links with other areas of mathematics e.g. coding theory.

Discrete Morse theory on infinite complexes

Jose A. Vilches
Universidad de Sevilla

We extend to the one-dimensional infinite case a theorem due to R. Forman which characterises discrete gradient fields on finite complexes.

TBA

Peter Wagner
University of Cambridge

Let s and t be integers satisfying $s \geq 2$ and $t \geq 2$. Let S be a tree of size s, and let P_t be the path of length t. I will show that, for every edge-colouring of the complete graph on n vertices, where $n = 224(s-1)^{2t}$, there is either a monochromatic copy of S or a rainbow copy of P_t. So, in particular, the number of vertices needed grows only linearly in t.

A brief introduction to Mono-Rainbow Ramsey numbers will also be given.

Consecutive choosability and a new graph invariant

Rob Waters
University of Nottingham

List colouring is a generalisation of ordinary graph colouring, in which the colour of each vertex must be chosen from a list of colours assigned to that vertex. We consider a variation of the list colouring problem, in which the lists are restricted to sets of consecutive integers, and the colours at adjacent vertices must differ by at least a fixed integer s.

For any graph G, we show that the ratio of the required list size to the separation s tends to a limit $\tau(G)$ as $s \to \infty$, which we call the *consecutive choosability ratio*. We obtain general bounds on $\tau(G)$ as well as exact values for various classes of graphs.

Semi-total colourings and the β parameter

Jini Williams
Open University

Let G be any graph and let μ be a semi-total colouring of G using $\Delta+1$ colours. A *beta edge* of G (with respect to μ) is an edge $e = wv$ such that $\mu(v) = \mu(w)$. Where β_μ is the number of such edges, and that

$$\beta = \min\{\beta_\mu : \mu \text{ a semi-total colouring of } G \text{ using } \Delta+1 \text{ colours}\}.$$

By a *near Type 1* graph we mean a connected graph G with $\chi'' > \Delta + 1$ such that $\chi''(G - e) = \Delta + 1$ for some edge e of G. (In particular, critical graphs are near Type 1.) The main result of this talk is that, for any near Type 1 graph,

$$\beta \leq \max\{\Delta, 2\Delta - 4\},$$

unless every total colouring of $G - e$ has distinct colours at v_1, v_2 and the spines at these vertices, and a certain sequence of Kempe chains exists, in which case we still have an upper bound for which will be presented.