Application of Resonance Perturbation Theory to Dynamics of Magnetization in Spin Systems Interacting with Local and Collective Bosonic Reservoirs

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Questions Addressed

Model

• N spins $1/2$, not directly interacting, coupled to local and collective bosonic heat reservoirs

• Each interaction has energy conserving and energy exchange channel

Program

• Start with a microscopic Hamiltonian description

Trace out all degrees of freedom but those of a single spin

Obtain **evolution of the reduced density matrix of a single spin**.

• Find **relaxation and dephasing rates** for single spin

• Find **evolution of total magnetization**

• Compare this evolution to the **Bloch equation**

Outline of Main Results

Single spin dynamics

We derive rigorous expression for reduced density matrix of single spin: main term describing relaxation and dephasing, plus remainder term small in couplings $homogeneously$ in time

Single spin relaxation

We show: Single-spin relaxation rate given by

$$\gamma_{\rm relax} = \frac{1}{4}\coth(\beta\omega/2)\left\{\lambda^2 J_c(\omega) + \mu^2 J_\ell(\omega)\right\}$$

ω : spin frequency

λ, μ : strengths of energy exchange collective and local couplings

$J_{c,\ell}(\omega)$: (collective, local) reservoir spectral densities

Only energy-exchange couplings contribute to this rate, effect of the local and the collective reservoirs the same

Single spin dephasing

We show: Single-spin dephasing rate given by

$$\gamma_{\mathrm{deph}} = \frac{1}{2}\gamma_{\mathrm{relax}} + \gamma_{\mathrm{cons}} + \gamma'$$

• γ_{cons} : contribution from energy conserving local and collective interactions, determined by spectral density at zero frequency

• γ' : effect on dephasing of a single spin due to all other spins

[Time-dependence of single spin off-diagonal density matrix elements is complicated, has not exponentially decaying contribution coming from the collective coupling; γ' defined to be the reciprocal of time by which that quantity is reduced to half its initial value]

• Explicit expression of γ' not simple

• $r = \frac{\text{collective coupling}}{\text{local coupling}} << 1 \Rightarrow \gamma' = O(r^2)$, indep. of N

• Large collective coupling $\Rightarrow \gamma' \sim \mathrm{const.}\gamma_{\mathrm{relax}}$, for constant indep. of N

Evolution of magnetization

Spins in homogeneous static magentic field pointing in z-direction

• We show: z-component of total magnetization vector relaxes to equilibrium value at *single-spin* relaxation rate γ_{relax}

$\longrightarrow$ In accordance with Bloch equation

• We show: Due to collective coupling, transverse total magnetic field follows **modified Bloch equation** with **time-dependent dephasing time** ($T_2 = T_2(t)$) and **time-dependent effective magnetic field** $B_{z,\mathrm{eff}}(t)$

Renormalization of T_2: for large times, Bloch equation becomes stationary, with renormalized $T_2(\infty)$ time

$$\frac{1}{T_2(\infty)} = \frac{1}{2}\gamma_{\mathrm{relax}} + \gamma_{\mathrm{cons}} + (N-1)\gamma''$$

Small ratio r collective/local coupling strenghts: $\gamma'' = O(r^2)$

• $r \sim N^{-1/2}$: collective coupling gives finite renormalization of T_2

• $r \sim N^{-1/2-\epsilon}$: no collective effect is visible in dephasing

• $r \sim N^{-1/2+\epsilon}$: drastic reduction of T_2? Perturbation theory not applicable!

Two-species spin system, $N = N_A + B_B$

We show:

• z-component of magnetization of either species relaxes with single-spin relaxation time (associated to that species)

• Transverse magnetization of either species dephases following modified Bloch equation with time-dependent T_2-time and effective magnetic field

• For large times, T_2-time of species A approaches

$$\frac{1}{T_{2,A}(\infty)} = \frac{1}{2}\gamma_{\mathrm{relax},A} + \gamma_{\mathrm{cons},A} + (N_A - 1)\gamma_A'' + N_B\gamma_B'',$$

with $\gamma_A = O(r_A^2)$, $\gamma_B = O(r_B^2)$ for small ratio r_A, r_B of the collective and local coupling constants

• Total magnetization is sum of that of species A and B. It is the sum of two terms decaying (relaxing and dephasing) at different rates so we cannot associate to it a total relaxation time or a total dephasing time

Model

Total Hamiltonian

$$
\begin{aligned}
H &= -\hbar \sum_{n=1}^{N} \omega_n S_n^z + \sum_{n=1}^{N} H_{\mathrm{R}_n} + H_{\mathrm{R}} \\
&\quad + \sum_{n=1}^{N} \lambda_n S_n^x \otimes \phi_c(g_c) + \sum_{n=1}^{N} \kappa_n S_n^z \otimes \phi_c(f_c) \\
&\quad + \sum_{n=1}^{N} \mu_n S_n^x \otimes \phi_n(g_n) + \sum_{n=1}^{N} \nu_n S_n^z \otimes \phi_n(f_n)
\end{aligned}
$$

$\omega_n > 0$: frequency of spin n

$$
S^z = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \text{and} \quad S^x = \frac{1}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}
$$

- H_{R}: Hamiltonian of the bosonic collective reservoir

$$H_{\mathrm{R}} = \int_{\mathrm{R}^3} \hbar |k| a^*(k) a(k) \mathrm{d}^3 k$$

$a(k), a^*(k)$ Bosonic annihilation, creation operators: $[a(k), a^*(l)] = \delta(k-l)$

- H_{R_n}: same Hamiltonian but of n-th individual reservoir
- Bosonic field operator

$$\phi(h) = \frac{1}{\sqrt{2}} \int_{\mathrm{R}^3} \{h(k) a^*(k) + h(k)^* a(k)\} \mathrm{d}^3 k.$$

- Coupling constants $\lambda_n, \kappa_n, \mu_n, \nu_n$
- Reservoir spectral density

$$J_h(\omega) := \pi \omega^2 \int_{S^2} |h(\omega, \Sigma)|^2 \mathrm{d}\Sigma$$

[$\mathrm{Re}\, \widehat{C}_h(\omega) = J_h(\omega) \coth(\beta\omega/2)$, where $\widehat{C}_h(\omega)$ is Fourier transform of symmetrized correlation function $C_h(t) = \frac{1}{2}[\langle \phi(h) \mathrm{e}^{\mathrm{i}tH_{\mathrm{R}}} \phi(h) \mathrm{e}^{-\mathrm{i}tH_{\mathrm{R}}} \rangle_\beta + \langle \mathrm{e}^{\mathrm{i}tH_{\mathrm{R}}} \phi(h) \mathrm{e}^{-\mathrm{i}tH_{\mathrm{R}}} \phi(h) \rangle_\beta]$]

Assumptions

Unperturbed **Bohr energies**: energy differences of $H_{\rm spin} = -\hbar\sum_{n=1}^N \omega_n S_n^z$

$$e(\underline{\sigma},\underline{\tau}) = -\frac{\hbar}{2}\sum_{n=1}^N \omega_n(\sigma_n - \tau_n) \qquad \underline{\sigma} = (\sigma_1,\ldots,\sigma_N) \in \{-1,+1\}^N$$

Gap $\Delta :=$ smallest non-zero difference $|e(\underline{\sigma},\underline{\tau}) - e(\underline{\sigma}',\underline{\tau}')|$
α : size of biggest coupling constant (local/collective, conserving/exchange)

(A) Small couplings relative to N

$$N^2\alpha^2 \ll \Delta$$

– homogeneous field: $\Delta = \hbar\omega \Rightarrow \alpha \sim 1/N$
– equidistributed energies: $\Delta \sim N2^{-2N} \Rightarrow \alpha \sim {\rm e}^{-N}$ (!)
– condition needed in technical estimates; from heurisitc physical considerations would expect condition $\alpha_c^2 N \ll \omega$ and $\alpha_\ell \ll \omega$ (local, collective coupling constants), ω typical spin frequency

(B) Spin frequencies $\{\omega_n\}$ are uncorrelated

If $e(\underline{\sigma}, \underline{\tau}) = e(\underline{\sigma}', \underline{\tau}')$ then $\sigma_n - \tau_n = \sigma_n' - \tau_n'$ for all n.

– Breaks permutation symmetry ($\rightarrow$ easier mathematical analysis)
– Nearly *homogeneous magnetic field*: $\omega_n = \omega + \delta\omega_n$, fluctuation $\delta\omega_n$ (e.g. uniform distribution in some interval
– Physical quantities continuous in $\delta\omega_n$, so can take $\delta\omega_n \rightarrow 0$ in those quantities to get case of homogeneous magnetic field ($\omega_n = \omega$ constant)

(C) Regularity of form factors

h : any of the coupling functions f_c, g_c, f_n, g_n in the Hamiltonian H.

$$h(|k|, \Sigma) = |k|^p \mathrm{e}^{-|k|^m} h'(\Sigma) \qquad \text{(spherical coordinates)}$$

with $p = -1/2 + n$, $n = 0, 1, 2, \ldots$ and $m = 1, 2$, and where h' is any angular function.

Reduced Dynamics of Single Spin $\rho_t^{(1)}$

Exactly solvable model

- Energy-conserving local (ν_ℓ) and collective (κ_c) interactions only
- Homogeneous spins (each same)
- Initial state: product of identical single spin states, $0 \le p \le 1$ (for $|\!\uparrow\rangle$)

$$[\rho_t^{(1)}]_{21} = [\rho_0^{(1)}]_{21}\ \mathrm{e}^{-\mathrm{i}\omega t}\ \underbrace{\mathrm{e}^{-\nu_\ell^2\Gamma_\ell(t)-\kappa_c^2\Gamma_c(t)}}_{\text{decay}}\ \underbrace{\mathcal{C}(N,t)}_{\text{oscillation}}$$

collective effect encoded in

$$\mathcal{C}(N,t) = [p\mathrm{e}^{-\mathrm{i}\kappa_c^2 S(t)} + (1-p)\mathrm{e}^{\mathrm{i}\kappa_c^2 S(t)}]^{N-1}$$

Decoherence function: $\Gamma_{\ell,c}(t) \longrightarrow t\widetilde{J}_{\ell,c}(0)$ $\quad$ (t large) Spectral Density

Oscillation: $S(t) \longrightarrow at$, where $a = \frac{-1}{2}\,\mathrm{P.V.}\int_{\mathrm{R}^3} \frac{|f(p)|^2}{|p|}\mathrm{d}^3p$ $\quad$ Lamb Shift

$|\mathcal{C}(N,t)|$ oscillates between 0 and 1, frequency $\kappa_c^2|a|/\pi$

Switching on energy exchange interactions

• Local (μ_ℓ) and collective (ν_c) energy exchange interactions generate relaxation process of populations & modify dephasing rate

• System not explicitly solvable; achievement of *resonance perturbation theory*: isolate main term from remainders (coupling constants $\alpha << 1$) **homogeneously in time**

Relaxation process:

$$[\rho_t^{(1)}]_{11} = \underbrace{\frac{e^{\beta\omega/2}}{e^{-\beta\omega/2} + e^{\beta\omega/2}}}_{\text{equilibrium}} + \underbrace{e^{-t\gamma_{\text{relax}}}\left[p - \frac{e^{\beta\omega/2}}{e^{-\beta\omega/2} + e^{\beta\omega/2}}\right]}_{\text{approach to equilibrium}} + O(\alpha^2)$$

with

$$\gamma_{\text{relax}} = \frac{1}{4}\left[\lambda_c^2 J_c(\omega) + \mu_\ell^2 J_\ell(\omega)\right]\coth(\beta\omega/2)$$

Does not depend on number of spins N

Dephasing process:

$$[\rho_t^{(1)}]_{21} = [\rho_0^{(1)}]_{21}\ \mathrm{e}^{-\mathrm{i}\omega t}\ \mathrm{e}^{-\nu_\ell^2\Gamma_\ell(t)-\kappa_c^2\Gamma_c(t)}\ \underbrace{\mathrm{e}^{-\frac{t}{2}\gamma_{\mathrm{relax}}}\ \mathrm{e}^{\mathrm{i}tX}\ \mathcal{C}(N,t)}_{\text{additional decay and oscillation}} + O(\alpha^2)$$

$X \in \mathrm{R}$: 'Lamb shift' symmetric in both collective interactions (indep. N)

- $\mathcal{C}(N,0) = 1$
- $|\mathcal{C}(N,t)| \le \mathrm{e}^{(N-1)[-\gamma t + c']}$, where $\gamma \ge 0$ (depends on all interactions except conserving local), γ and $c' > 0$ indep. of N
- If the energy conserving collective coupling and at least one of the energy exchange couplings (local or collective) are nonzero, then $\gamma > 0$.
- $|\mathcal{C}(N,t)|$ decays to $1/2$ (half its initial value) *no later than* at time $1/\gamma'$,

$$\gamma' = \gamma \left[\frac{\ln 2}{N-1} + c'\right]^{-1} \qquad \sim \gamma/c' \ (N \text{ large})$$

• Total dephasing rate: $\gamma_{\rm deph} = \frac{1}{2}\gamma_{\rm relax} + \gamma_{\rm cons} + \gamma'$

$$\begin{aligned}
\gamma_{\rm relax} &= \frac{1}{4}\big[\lambda_c^2 J_c(\omega) + \mu_\ell^2 J_\ell(\omega)\big] \coth(\beta\omega/2) \\
\gamma_{\rm cons} &= \frac{1}{2\beta}\big[\kappa_c^2 \widetilde{J}_c(0) + \nu_\ell^2 \widetilde{J}_\ell(0)\big] \\
\gamma' &= \gamma \left[\frac{\ln 2}{N-1} + c'\right]^{-1}
\end{aligned}$$

• Behaviour of γ': Set (κ_c cons coll, λ_c exch coll, μ_ℓ exch local)

$$r = \frac{\kappa_c^2}{\lambda_c^2 + \mu_\ell^2}$$

• Collective coupling weak: $r \sim 0 \Rightarrow \gamma' \sim \mathrm{const.} r|\kappa_c|$, with const. indep of N.

• Collective coupling strong: $r \sim 1 \Rightarrow \gamma' \sim \mathrm{const.} \lambda_c^2 J_c(\omega)$, with const. indep of N.

Evolution of Magnetization

Total spin operator: $\vec{S} = \begin{bmatrix} S^x \\ S^y \\ S^z \end{bmatrix}, \qquad S^{x,y,z} = \sum_{j=1}^{N} S_j^{x,y,z}$

Longitudinal component: S^z (direction of static external magnetic field)
Transverse component: $S_j^- = S_j^x - \mathrm{i} S_j^y$

Homogeneous magnetic field $\vec{B} = -\omega \vec{e}_z$

Longitudinal magnetization component: Resonance method ⇒

$$\langle S^z \rangle_t = \frac{N}{2} \tanh(\beta\omega/2)[1 - \mathrm{e}^{-t\gamma_{\mathrm{relax}}}] + \mathrm{e}^{-t\gamma_{\mathrm{relax}}} \langle S^z \rangle_0 \; + O(\alpha^2)$$

$\langle \cdot \rangle_0$: initial spin state, product of identical single-spin states

$\gamma_{\mathrm{relax}} = \frac{1}{4}\left[\lambda_c^2 J_c(\omega) + \mu_\ell^2 J_\ell(\omega)\right] \coth(\beta\omega/2)$ single-spin relaxation rate

Total longitudinal magnetization relaxes to equilibrium value at single-spin relaxation rate

Above expression is time-integrated version of **Bloch equation for longitudinal magnetization component** for homogeneous magnetic field $\vec{B} = -\omega \vec{e}_z$,

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle S^z \rangle_t = -\frac{1}{\tau_{\mathrm{relax}}}\big[\langle S_j^z \rangle_t - \frac{N}{2}\tanh(\beta\omega/2)\big]$$

T_1 time is $T_1 = \tau_{\mathrm{relax}} = 1/\gamma_{\mathrm{relax}}$ and does not depend on collective effects, nor on number of spins

Transverse magnetization component: Resonance method $\Rightarrow$

$$\langle S^-\rangle_t = \mathrm{e}^{-\mathrm{i}t(\omega - X)} \mathrm{e}^{-t[\frac{1}{2}\gamma_{\mathrm{relax}} + \gamma_{\mathrm{cons}}]} \underbrace{[\mathcal{D}(t)]^{N-1}}_{\text{collective effect}} \langle S^-\rangle_0 + O(\alpha^2)$$

X : single-spin 'Lamb shift' contribution

γ_{relax}, γ_{cons} : single-spin decays

"Ordinary" transverse Bloch equation would read

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle S^-\rangle_t = -\frac{1}{T_2}\langle S^-\rangle_t + \mathrm{i}B_z\langle S^-\rangle_t$$

Differentiating the true $\langle S^-\rangle_t$ get **modifed Bloch equation**

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\langle S^-\rangle_t &= -\Gamma(t)\langle S^-\rangle_t + \mathrm{i}B(t)\langle S^-\rangle_t \\
\Gamma(t) &= \frac{1}{2}\gamma_{\mathrm{relax}} + \gamma_{\mathrm{cons}} - (N-1)\,\mathrm{Re}\frac{\mathrm{d}}{\mathrm{d}t}\ln\mathcal{D}(t) \\
B(t) &= -\omega + X + (N-1)\,\mathrm{Im}\frac{\mathrm{d}}{\mathrm{d}t}\ln\mathcal{D}(t)
\end{aligned}$$

$\mathcal{D}(t)$: single-spin quantity (indep. of N), decay & oscillations

Transverse magnetization satisfies modified Bloch equation, where T_2-time and effective magnetic field are **time-dependent**.

We have $T_2 = 1/\Gamma(t)$, $B_{z,\mathrm{eff}} = B(t)$, with

$$\begin{aligned}\Gamma(t) &= \tfrac{1}{2}\gamma_{\mathrm{relax}} + \gamma_{\mathrm{cons}} - (N-1)\,\mathrm{Re}\tfrac{\mathrm{d}}{\mathrm{d}t}\ln\mathcal{D}(t)\\ B(t) &= -\omega + X + (N-1)\,\mathrm{Im}\tfrac{\mathrm{d}}{\mathrm{d}t}\ln\mathcal{D}(t)\end{aligned}$$

Both time-dependencies have factor N: **collective effects**

Deviation of "static" Bloch equation given by $\frac{\mathrm{d}}{\mathrm{d}t}\ln\mathcal{D}(t)$

$r = \frac{\kappa_c^2}{\mu_\ell^2} << 1$ (energy exch. coll. weak rel. to energy exch. local) $\Rightarrow$

$$\left|\frac{\mathrm{d}}{\mathrm{d}t}\ln\mathcal{D}(t)\right| \le C|r|, \quad \lim_{t\to\infty}\frac{\mathrm{d}}{\mathrm{d}t}\ln\mathcal{D}(t) = 4\mathrm{i}r\frac{\tanh(\beta\omega/2)}{1-\mathrm{e}^{-\beta\omega}}\gamma_{\mathrm{relax}} + O(r^2)$$

For small collective coupling $T_2(t)$ and $B(t)$ stabilize as $t \to \infty$,

$$T_2(\infty)^{-1} = \frac{1}{2}\gamma_{\rm relax} + \gamma_{\rm cons} + (N-1)\gamma'', \qquad \gamma'' = O(r^2)$$

• $r \sim N^{-1/2} \Rightarrow$ finite renormalization of T_2-time

• $r \sim N^{-1/2-\epsilon}$ (any $\epsilon > 0$) $\Rightarrow$ no collective effect visible in dephasing

• $r \sim N^{-1/2+\epsilon}$ (any $\epsilon > 0$) above expression suggests that collective interaction may decrease T_2-time drastically for large N. But perturbation theory not valid in this regime!

Multi-species inhomogeneity

• N spins grouped into two classes A, B characteized by different properties ω_A, ω_B, etc.

• Relative sizes $N = N_A + N_B$

• Relative magnetization

$$\vec{S}_A = \sum_{j \text{ in class } A} \vec{S}_j$$

• Resonance method $\Rightarrow$ (modulo $O(\lambda^2)$ terms)

$$\begin{aligned} \langle S_A^z \rangle_t &= \frac{N_A}{2} \tanh(\beta\omega_A/2)[1 - \mathrm{e}^{-t\gamma_{\text{relax},A}}] + \mathrm{e}^{-t\gamma_{\text{relax},A}} \langle S_A^z \rangle_0 \\ \langle S_A^- \rangle_t &= \mathrm{e}^{-\mathrm{i}t(\omega_A - X_A)} \mathrm{e}^{-t[\frac{1}{2}\gamma_{\text{relax},A} + \gamma_{\text{cons},A}]} [\mathcal{D}_A(t)]^{N_A - 1} [\mathcal{D}_B(t)]^{N_B} \langle S_A^- \rangle_0 \end{aligned}$$

Relaxation: single-spin rate $\gamma_{\text{relax},A}$

Dephasing: modified Bloch equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle S_A^- \rangle_t = -\Gamma_A(t)\langle S_A^- \rangle_t + \mathrm{i}B_A(t)\langle S_A^- \rangle_t$$

with

$$\begin{aligned}\Gamma_A(t) &= \frac{1}{2}\gamma_{\mathrm{relax},A} + \gamma_{\mathrm{cons},A}\\ &\quad -(N_A - 1)\,\mathrm{Re}\frac{\mathrm{d}}{\mathrm{d}t}\ln\mathcal{D}_A(t) - N_B\,\mathrm{Re}\frac{\mathrm{d}}{\mathrm{d}t}\ln\mathcal{D}_B(t)\\ B_A(t) &= -\omega_A + X_A + (N_A - 1)\,\mathrm{Im}\frac{\mathrm{d}}{\mathrm{d}t}\ln\mathcal{D}_A(t) + N_B\mathrm{Im}\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{D}_B(t)\end{aligned}$$

Renormalization of dephasing time (weak coll. coupling and large times):

$$\Gamma_A(t) \to T_{2,\mathrm{A}}(\infty)^{-1} = \frac{1}{2}\gamma_{\mathrm{relax},A} + \gamma_{\mathrm{cons},A} + (N_A - 1)\gamma''_A + N_B\gamma''_B$$

with $\gamma_{A,B} = O(r^2_{A,B})$

– Total magnetization: $\langle S \rangle_t = \langle S_A \rangle_t + \langle S_B \rangle_t$

– z-component relaxes as sum of two exponentially decaying quantities with different rates (corresponding to A and B); cannot associate to it a single decay rate

– Total transverse magnetization is sum of that of species A and B; each contribution evolves according to its modified Bloch equation; for large times dephasing time approaches renormalized constant value: sum of two terms decaying at different rates; total transverse magnetization does not have single decay rate

Outline of Resonance Method

1. Hilbert space representation (GNS)

General density matrix: $\rho = \sum_n p_n |\psi_n\rangle\langle\psi_n|$, $\psi_n \in \mathcal{H}$

Identify: $|\psi\rangle\langle\chi| \mapsto \psi \otimes \chi^* \in \mathcal{H} \otimes \mathcal{H}$

Density matrix represented as vector in *new* Hilbert space:

$$\rho \mapsto \sum_n p_n \psi \otimes \psi^* \quad \in \mathcal{H} \otimes \mathcal{H}$$

$$\langle A \rangle = \mathrm{Tr}_{\mathcal{H}}(\rho A) = \langle \Omega, (A \otimes \mathbb{1})\Omega \rangle_{\mathcal{H}\otimes\mathcal{H}}$$

with

$$\Omega = \sum_n \sqrt{p_n}\psi_n \otimes \psi_n^*$$

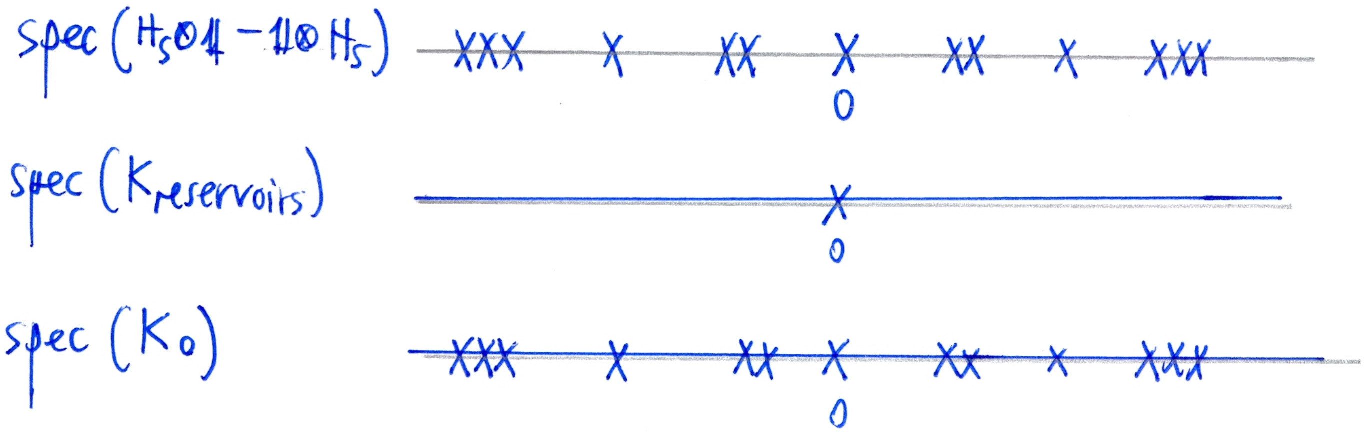
N spins plus local and collective reservoirs $\Rightarrow \mathcal{H}, \Omega$

$$\langle A \rangle_t = \langle \Omega, \mathrm{e}^{\mathrm{i}tK}(A \otimes \mathbb{1})\Omega \rangle_{\mathcal{H}\otimes\mathcal{H}} \qquad K : \text{"Liouville operator"}$$

2. Spectral analysis of K and dynamics

$$K = K_0 + \alpha V$$
$$K_0 = H_{\mathrm{spins}} \otimes \mathbb{1}_{\mathrm{spins}} - \mathbb{1}_{\mathrm{spins}} \otimes H_{\mathrm{spins}} + K_{\mathrm{reservoirs}}$$

α : interaction parameter, V : interaction operator



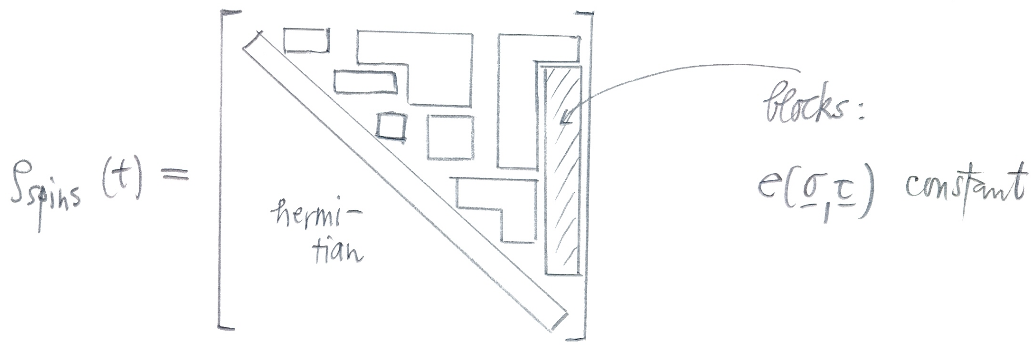
Meaning of eigenvalues $e(\underline{\sigma},\underline{\tau})$: under *free dynamics* ($\alpha = 0$)

$$\langle\underline{\sigma}|\rho_{\mathrm{spins}}(t)|\underline{\tau}\rangle = \mathrm{e}^{-ite(\underline{\sigma},\underline{\tau})}\langle\underline{\sigma}|\rho_{\mathrm{spins}}(0)|\underline{\tau}\rangle$$

What happens as $\alpha \neq 0$?

• matrix elements of ρ_{spin} do not evolve independently any longer (but in "clusters")

• dispersive reservoirs induce irreversibility: $e(\underline{\sigma}, \underline{\tau})$ become *complex energies* $\varepsilon(\underline{\sigma}, \underline{\tau})$ $(\mathrm{Im}\varepsilon$: decay rates)

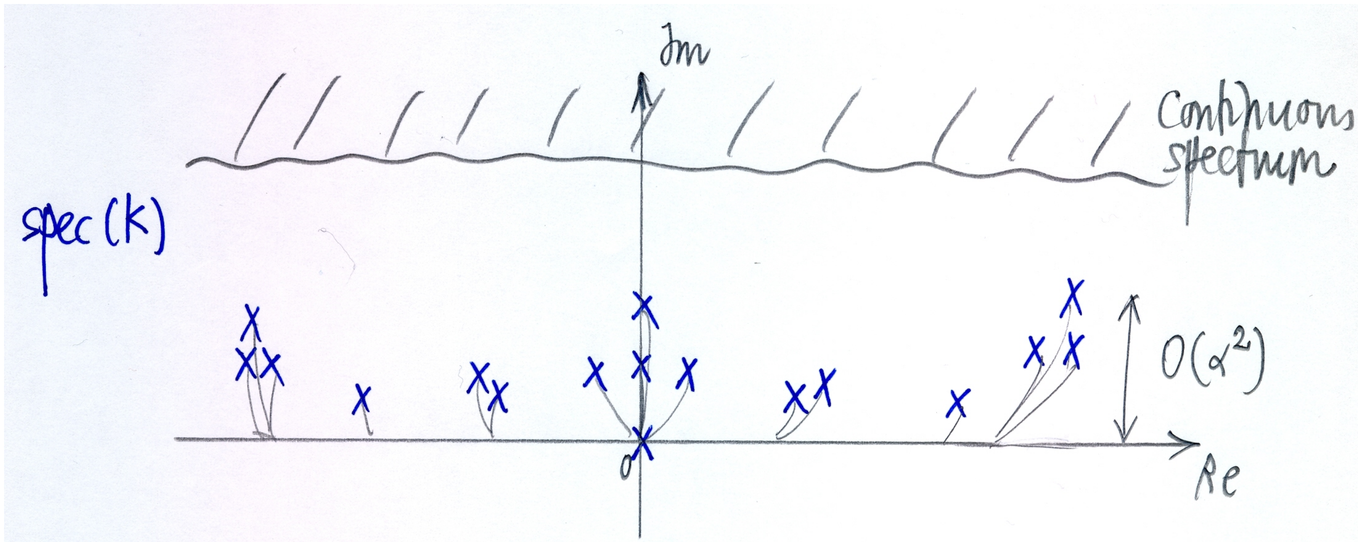


Within each block:

$$\langle \underline{\sigma} | \rho_{\mathrm{spin}}(t) | \underline{\tau} \rangle = \sum_{\underline{\sigma}', \underline{\tau}' \text{ in block}} A_t(\underline{\sigma}, \underline{\tau}; \underline{\sigma}', \underline{\tau}') \ \langle \underline{\sigma}' | \rho_{\mathrm{spin}}(0) | \underline{\tau}' \rangle + O(\alpha^2)$$

$$A_t(\underline{\sigma}, \underline{\tau}; \underline{\sigma}', \underline{\tau}') = \sum_{s=1}^{\mathrm{mult}\ e(\underline{\sigma}, \underline{\tau})} \mathrm{e}^{\mathrm{i}t\varepsilon_e^{(s)}} C(s)$$

spectrum of K as $\alpha \neq 0$



$$K \to \sum_{e,s} \mathrm{e}^{\mathrm{i}t\varepsilon_e^{(s)}} |\xi_e^{(s)}\rangle\langle\widetilde{\xi}_e^{(s)}| + O(\alpha^2)$$

Averages become

$$\langle A\rangle_t = \sum_{e,s} \mathrm{e}^{\mathrm{i}t\varepsilon_e^{(s)}} \langle\Omega|\xi_e^{(s)}\rangle\langle\widetilde{\xi}_e^{(s)}|\ (A\otimes \mathbb{1})\ |\Omega\rangle\ + O(\alpha^2)$$

Analysis of structure of ξ, ε gives final result