Dark Matter Spin-Spin Interaction through the Pseudo-Scalar Vacuum Field

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We suggest that the pseudo-scalar vacuum field (PSV) in the dark matter (DM) sector of the Universe may be as important as the electromagnetic vacuum field in the baryonic sector. In particular, the spin-spin interaction between the DM fermions, mediated by PSV, may represent the strongest interaction between the DM fermions due to the absence of the electric charge and the magnetic dipole moment. Based on this assumption, we consider the influence of the spin-spin interaction, mediated by PSV, on the spin precession of the DM fermions (e. g. neutralino). In the secular approximation, we obtain the exact expression describing the frequency of the precession and estimate the decoherence rate.

Keywords: dark matter, axion, neutralino, spin-spin interaction

INTRODUCTION

It is well-known that the pseudo-scalar vacuum (PSV) field (e.g. axionic field) interacts with the spins of the baryonic matter (see, for example, [1, 2]). As a result, PSV mediates the spin-spin interactions in the baryonic matter (see, for example, [3]). Certainly, interactions between the PSV and baryonic matter represent a tiny correction to the electromagnetic interactions. We suggest that in the dark sector of the Universe the situation may be opposite: due to the absence of the electric charge and the magnetic moment, the interaction between the dark matter (DM) fermionic spins (e.g. neutralino spins) and PSV may manifest the leading interaction in DM. (We note here that neutralino remains one of the main candidates for DM [4–8]. However, there are many other fermionic candidates as well - see, for example, [9, 10].) In particular, the DM fermionic spins could produce permanent fields similar to the magnetic fields in the baryonic sector.

Based on this assumption we analyze precession of the two DM fermionic spins, coupled through PSV, in a non-uniform external field produced by the other DM fermions. We assume that the two DM spins are located at the points with the position vectors, $\mathbf{r}_1$ and $\mathbf{r}_2$. The frequencies of the spin precession in the external axion field are given by ε_1 and ε_2. In order to obtain the analytical expressions describing the spin precession, we assume that ε_1, ε_2 and $|\varepsilon_1 - \varepsilon_2|$ are larger than the interaction between the axion field and spins. This allows us to use the secular approximation well-known in magnetic resonance (see, for example, [11]).

Note, that we limited ourselves by considering the consequences of the interaction of the DM fermionic spins through the PSV. So, we do not consider many other very important and widely discussed problems such as chiral anomaly of fermions [12], the issues related to ultra-light axions [13, 14], many existing protocols for axion and neutralino detection [15–17], etc.

In this paper, we analyze precession of the two DM fermionic spins 1/2, coupled through PSV, in a non-uniform external axion field ($\sim \nabla\varphi(\mathbf{r})$) produced by the other DM fermions or by axion-originated topological defects. The frequencies of the spin precession in the external axion field are given by: $\varepsilon_{1,2} = 2\lambda|\nabla\varphi(\mathbf{r}_{1,2})|/\hbar$. It is important to note that: (i) the frequencies, $\varepsilon_{1,2}$ are analogous to the well-known nuclear magnetic resonance (NMR) or electron paramagnetic resonance (EPR) frequencies of precession of single nuclear or electron spin in the permanent magnetic field, (ii) these frequencies are proportional to the first order of the coupling constant, λ, and (iii) the search of axions, discussed in [1, 2], is based, in particular, on detection of precession frequencies of electron or nuclear spins in axion field.

In our paper, we are interested in a different effect, namely, in the interaction of two DM spins through the PSV. We demonstrate that in this case, one spin rotates around the other non-rotating spin due to their interaction through the PSV. So, this effect is (i) of the second order in λ, (ii) analogous to the well-known Lamb shift of energy levels in the vacuum electromagnetic field, and (iii) has no direct relation to the $\varepsilon_{1,2}$ spin precession.

The interaction of the two spins with the permanent field and PSV can be written as (see, for example, [1, 2]),

$$H_{int} = \lambda \sum_{\alpha=1,2} \nabla\varphi(\mathbf{r}_\alpha) \cdot \boldsymbol{\sigma}^\alpha. \tag{1}$$

Here φ, as above, is the PSV, λ is the coupling constant, and $\boldsymbol{\sigma}^\alpha \equiv (\sigma_x^\alpha, \sigma_y^\alpha, \sigma_z^\alpha)$ is the vector built by Pauli matrices of spin α, located at $\mathbf{r}_\alpha$.

Through this paper, we will use the natural units convention, $\hbar = c = 1$.

PRECESSION OF THE SPINS

Our total Hamiltonian, H, describes, in the secular approximation, the two DM fermionic spins interacting with the external field, oriented in the z-direction (direction of the $\nabla\varphi$), and the PSV:

$$\begin{aligned} H =& \frac{1}{2} \sum_{\alpha=1,2} \varepsilon_\alpha \sigma_z^\alpha + \sum_{\mathbf{k}} \omega_k \hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}} \\ &+ \frac{i\lambda}{\sqrt{V}} \sum_{\alpha=1,2} \sigma_z^\alpha \sum_{\mathbf{k}} \frac{k_z}{\sqrt{2\omega_k}} \big(\hat{a}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}_\alpha} - \hat{a}_{\mathbf{k}}^\dagger e^{-i\mathbf{k}\cdot\mathbf{r}_\alpha}\big). \end{aligned} \tag{2}$$

Here ε_α is the transition frequency of spin α in the external permanent field, $\omega_k = \sqrt{k^2 + m^2}$ is the frequency of the field mode with wave number $\mathbf{k}$, where m is the mass of a DM boson, and V is the quantization volume.

We will consider the initial state, $|\psi\rangle$, of the spin-PSV system as the tensor product of the spin $|\psi_s\rangle$ and the PSV $|\psi_V\rangle$ states: $|\psi\rangle = |\psi_s\rangle \otimes |\psi_V\rangle$. Below, we use the σ_z-representation for the single spin states: vectors $|0_\alpha\rangle$ and $|1_\alpha\rangle$ ($\alpha = 1, 2$) denote the spin pointing in the negative and positive z-direction, respectively. For two spins, we use the basis: $|0\rangle \equiv |0_2 0_1\rangle$, $|1\rangle \equiv |0_2 1_1\rangle$, $|2\rangle \equiv |1_2 0_1\rangle$, $|3\rangle \equiv |1_2 1_1\rangle$. Then, the initial spin wave function is, $|\psi_s\rangle = \sum_{i=0}^{3} C_i |i\rangle$, $(\sum_{i=0}^{3} |C_i|^2 = 1)$. The corresponding initial spin density matrix is,

$$\rho_s(0) = \sum_{i,j=0}^{3} \rho_{ij}(0) |i\rangle\langle j|, \ (\rho_{ij}(0) = C_i C_j^*). \tag{3}$$

The concrete values of the amplitudes, C_i, are not important for us. In the interaction representation, the evolution operator of the spin-PSV system can be written as [18],

$$U(t) = \hat{T} \exp\Big(-\frac{i}{\hbar} \int_0^t dt' H_{int}(t')\Big) = \exp\Big(\frac{i}{2} \sum_{\alpha,\beta=1,2} \sigma_z^\alpha \sigma_z^\beta \nu_{\alpha\beta}(t)\Big) \exp\Big(\sum_{\alpha=1,2} \sigma_z^\alpha \sum_{k} \big(e^{-i\mathbf{k}\cdot\mathbf{r}_\alpha} \xi_k(t) \hat{a}_{\mathbf{k}}^\dagger - e^{i\mathbf{k}\cdot\mathbf{r}_\alpha} \xi_k^*(t) \hat{a}_{\mathbf{k}}\big)\Big), \tag{4}$$

where

$$\xi_k(t) = \frac{\lambda k_z (e^{i\omega_k t} - 1)}{\sqrt{2V} \omega_k^{3/2}}, \tag{5}$$

and

$$\nu_{\alpha\beta} = i \sum_{\mathbf{k}} \cos(\mathbf{k} \cdot \mathbf{r}_{\alpha\beta}) \int_0^t dt' \big(\xi_k(t') \dot{\xi}_k^*(t') - \xi_k^*(t') \dot{\xi}_k(t')\big). \tag{6}$$

Here $\mathbf{r}_{\alpha\beta} = \mathbf{r}_\beta - \mathbf{r}_\alpha$.

Using the evolution operator $U(t)$ of Eq. (4), one can write the total density matrix as,

$$\rho_{tot}(t) = U(t) \rho_{tot}(0) U^{-1}(t), \tag{7}$$

where $\rho_{tot}(0) = |\psi\rangle\langle\psi|$. Next, we have obtained the 4×4 reduced spin density matrix, $\rho_s(t)$, by tracing out the PSV degrees of freedom. (The details of computations are given in Appendix A.)

Using Eqs. (3) and (7), we calculate the x-component of the total spin:

$$\begin{aligned} \bar{S}_x(t) =& \sum_{\alpha=1,2} Tr(\rho_s(t) \sigma_x^\alpha) = \sum_{\alpha=1,2} \sum_{i=0}^{3} \langle i | \rho_s(t) \sigma_x^\alpha | i \rangle \\ =& \rho_{01}(t) + \rho_{02}(t) + \rho_{13}(t) + \rho_{23}(t) + c.c., \end{aligned} \tag{8}$$

where $\rho_{ij}(t)$ are the time-dependent components of the reduced spin density matrix. One can see that $\bar{S}_x(t)$ is associated with the precession of a single spin while the other spin remains in a basis state.

For definiteness, consider the element $\rho_{01}(t)$ of the reduced spin density matrix. We have derived the following expression:

$$\rho_{01}(t) = \rho_{01}(0)e^{i\delta(t)-\gamma(t)}, \tag{9}$$

where,

$$\delta(t) =-2\big(\nu_{12}(t) - \sum_{\mathbf{k}} \sin(\mathbf{k}\cdot\mathbf{r}_{12})|\xi_k(t)|^2\big), \tag{10}$$

$$\gamma(t) =2\sum_{\mathbf{k}} |\xi_k(t)|^2. \tag{11}$$

The functions, $\delta(t)$ and $\gamma(t)$, describe the frequency shift and decoherence of the spin precession due to the indirect interaction between the fermionic spins through the PSV. Substituting the expression (5) for $\xi_k(t)$ and changing sums to integrals, $(1/V)\sum_{\mathbf{k}} \to (1/(2\pi)^3)\int d^3k$, we obtain:

$$\begin{aligned}\delta(t) =&- \frac{\lambda^2}{4\pi^3}\int d^3k\bigg(\frac{k_z^2\cos(\mathbf{k}\cdot\mathbf{r}_{12})(\omega_k t - \sin(\omega_k t))}{\omega_k^3}\\ &- \frac{2k_z^2\sin(\mathbf{k}\cdot\mathbf{r}_{12})\sin^2(\omega_k t/2)}{\omega_k^3}\bigg),\end{aligned} \tag{12}$$

$$\gamma(t) =\frac{\lambda^2}{4\pi^3}\int \frac{d^3k k_z^2\sin^2(\omega_k t/2)}{\omega_k^3}. \tag{13}$$

The frequency shift. – We computed the 3D integral in Eq. (12), assuming that $mL \ll 1$, where $L = |\mathbf{r}_{12}|$, (see Appendix B for details), and obtained the following expression for the phase shift:

$$\delta(t) = \omega_s t,\ \omega_s = \frac{\lambda^2}{\pi}\cdot\frac{3\cos^2\theta - 1}{r_{12}^3}. \tag{14}$$

Here ω_s is the frequency shift, θ is the polar angle of the vector $\mathbf{r}_{12}$ connecting the two spins. Formula (14) is valid for $t > r_{12}$; for $t < r_{12}$ we have $\delta(t) = 0$.

One can see that the phase shift is proportional to time. The coefficient at t is the frequency shift, ω_s, which is proportional to $(3\cos^2\theta - 1)/r_{12}^3$. This factor is similar to that obtained for the magnetic dipole-dipole interaction in the baryonic matter [11], in spite of the magnetic moment for DM fermion is zero. In both cases the frequency shift changes its sign at the magic angle, $\cos^2\theta = 1/3$. Thus, we come to the conclusion that the spin-spin fermionic interaction, mediated by PSV, is similar to the magnetic dipole-dipole interaction mediated by the vacuum electromagnetic field.

Decoherence. – The integral in Eq. (13), describing the decoherence, is diverging. This divergence is non-physical. It can be eliminated with proper renormalizations. In Appendix A we compute this integral using the dimensional regularization.

Generation of entanglement entropy. – By tracing out the pseudo-scalar degrees of freedom, we generate the entanglement entropy (EE), $E(t) = -Tr(\rho_s(t)\ln\rho_s(t))$, in the spin sub-system. The EE is created independently of the initial spin wave function (with $E(0) = 0$) disentangled or entangled. Namely, in both cases, the trace over the pseudo-scalar vacuum field creates entanglement for spin sub-system, at $t > 0$ ($E(t) > 0$).

CONCLUSION

We suggested that in the dark sector of the Universe the interaction between the PSV of the DM bosons and DM fermions may be as important as the electromagnetic interaction in the baryonic sector. In particular, the spin-spin interaction between the DM fermions, mediated by PSV, may represent the leading interaction in DM.

Based on this assumption, we consider the following situation in the dark sector of the Universe (below, for definiteness, we will write neutralinos and axions instead of DM fermions and bosons). In the first scenario, a large ensemble of neutralino spins (ENS) produces a strong permanent axionic field. This is similar to the permanent magnetic field produced by a large ensemble of electron spins of a permanent magnet in the baryonic sector. In the second scenario, the permanent axionic field is produced by axion topological defects of soliton or domain wall types [1, 2].

Next, we consider the system of two neutralino spins which experience (i) the permanent axionic field produced by the ENS or by axion-generated topological defects and (ii) indirect interaction between themselves through the axion vacuum field. Each neutralino spin precesses in the permanent axionic field like nuclear or electron spin precesses in the magnetic field of the permanent magnet. The frequency of this precession is proportional to the first order in the perturbation constant, λ, and is a subject of intensive experimental research for axions by using electron or nuclear spins. (See, for example, [1, 2], and references therein.) Note that the neutralino spin precession would generate radiation of the real axions which could be detected experimentally like electron spin precession generates radiation of the real photons detected by the NMR and EPR techniques. (In this work we do not explore the process of radiation of real axions).

Our main attention is concentrated on an indirect interaction between the two neutralino spins mediated by the vacuum axionic field like the magnetic dipole-dipole interaction between the electron spins is mediated by the electromagnetic vacuum. The indirect spin-spin interaction is the only non-gravitational interaction between the neutralinos as these particles do not have an electric charge and the magnetic moment. In this work, we have studied the influence of the indirect interaction between the neutralino spins on the spin precession. In the secular approximation, we have derived exact analytical expressions for the frequency shift and decoherence rate of the neutralino spin precession caused by the interaction mediated by the vacuum axionic field. We demonstrate that this frequency shift is analogous to the Lamb shift of energy levels in electromagnetic vacuum, and it is proportional to the second order in the perturbation constant, λ. The mechanism of this frequency shift is related to the precession of one spin around the other spin, due to their interaction through the axion vacuum field. This mechanism of spin precession has no direct relation to the spin precession discussed in recent proposals (see [1, 2], and references therein). Certainly, our results are valid not only for neutralinos and axions but for any DM fermions interacting with the PSV of the DM bosons.

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Supplemental Material

Appendix A

Evolution operator

In this section, we derive the elements of the reduced spin density matrix. First, consider the evolution operator written as,

$$U(t) = \exp\Big(\frac{i}{2}\sum_{\alpha,\beta}\sigma_z^\alpha\sigma_z^\beta\nu_{\alpha\beta}(t)\Big)D(\xi_k(t)),\ \alpha,\beta=1,2, \tag{15}$$

where $D(\xi_k)$ is the displacement operator:

$$D(\xi_k(t)) = \exp\Big(\sum_{\alpha,\mathbf{k}}\sigma_z^\alpha\big(e^{-i\mathbf{k}\cdot\mathbf{r}_\alpha}\xi_k(t)\hat{a}^\dagger_{\mathbf{k}} - e^{i\mathbf{k}\cdot\mathbf{r}_\alpha}\xi_k^*(t)\hat{a}_{\mathbf{k}}\big)\Big). \tag{16}$$

Here,

$$\xi_k(t) = \frac{\lambda k_z(e^{i\omega_k t}-1)}{\sqrt{2V}\omega_k^{3/2}}, \tag{17}$$

$$\nu_{\alpha\beta} = i\sum_{\mathbf{k}}\cos(\mathbf{k}\cdot\mathbf{r}_{\alpha\beta})\int_0^t dt'\big(\xi_k(t')\dot{\xi}_k^*(t') - \xi_k^*(t')\dot{\xi}_k(t')\big), \tag{18}$$

and we set, $\mathbf{r}_{\alpha\beta} = \mathbf{r}_\beta - \mathbf{r}_\alpha$.

The displacement operator can be recast as,

$$D(\xi_k) = e^{-\kappa(t)/2}e^{\sum_\alpha\sigma_z^\alpha e^{-i\mathbf{k}\cdot\mathbf{r}_\alpha}\xi_k\hat{a}^\dagger_{\mathbf{k}}}e^{-\sum_\alpha\sigma_z^\alpha e^{i\mathbf{k}\cdot\mathbf{r}_\alpha}\xi_k^*\hat{a}_k}, \tag{19}$$

where $\kappa(t) = \sum_{\alpha,\beta}\mu_{\alpha\beta}(t)\sigma_z^\alpha\sigma_z^\beta$ and $\mu_{\alpha\beta}(t) = e^{i\mathbf{k}\cdot\mathbf{r}_{\alpha\beta}}|\xi_k(t)|^2$. Taking into account that $\hat{a}_{\mathbf{k}}|\Psi_V\rangle = 0$, we obtain,

$$D(\xi_k)|\Psi_V\rangle = e^{-\kappa(t)/2}\sum_m\frac{(\sum_\alpha\sigma_z^\alpha e^{-i\mathbf{k}\cdot\mathbf{r}_\alpha}\xi_k)^m}{m!}(\hat{a}^\dagger_{\mathbf{k}})^m|\Psi_V\rangle. \tag{20}$$

Next, using the relation, $(\hat{a}^\dagger_{\mathbf{k}})^m|\Psi_V\rangle = \sqrt{m!}\,|m_{\mathbf{k}}\rangle$, we find,

$$D(\xi_k)|\Psi_V\rangle = e^{-\kappa(t)/2}\sum_m\frac{(\sum_\alpha\sigma_z^\alpha e^{-i\mathbf{k}\cdot\mathbf{r}_\alpha}\xi_k)^m}{\sqrt{m!}}|m_{\mathbf{k}}\rangle. \tag{21}$$

Before proceeding further, it is convenient to introduce a new auxiliary displacement operator:

$$D(\xi_k^a) = e^{\xi_k^a\hat{a}^\dagger_{\mathbf{k}} - \xi_k^{a*}\hat{a}_{\mathbf{k}}} = e^{-|\xi_k^a|^2/2}e^{\xi_k^a\hat{a}^\dagger_{\mathbf{k}}}e^{-\xi_k^{a*}\hat{a}_{\mathbf{k}}}, \tag{22}$$

where $\xi_k^a = \kappa_a\xi_k$ $(a=1,2)$, with $\kappa_1 = \sum_\alpha e^{-i\mathbf{k}\cdot\mathbf{r}_\alpha}$ and $\kappa_2 = \sum_\alpha(-1)^\alpha e^{-i\mathbf{k}\cdot\mathbf{r}_\alpha}$.

Now, applying the evolution operator to the basis states, we obtain,

$$U|0\rangle\otimes|\Psi_V\rangle = e^{i(\nu+\nu_{12})}|0\rangle\otimes\prod_k D\big(-\xi_k^1\big)|\Psi_V\rangle, \tag{23}$$

$$U|1\rangle\otimes|\Psi_V\rangle = e^{i(\nu-\nu_{12})}|1\rangle\otimes\prod_k D\big(-\xi_k^2\big)|\Psi_V\rangle, \tag{24}$$

$$U|2\rangle\otimes|\Psi_V\rangle = e^{i(\nu-\nu_{12})}|2\rangle\otimes\prod_k D\big(\xi_k^2\big)|\Psi_V\rangle, \tag{25}$$

$$U|3\rangle\otimes|\Psi_V\rangle = e^{i(\nu+\nu_{12})}|3\rangle\otimes\prod_k D\big(\xi_k^1\big)|\Psi_V\rangle, \tag{26}$$

where $\nu = (1/2)(\nu_{11}+\nu_{22})$.

The matrix elements of the reduced density matrix are given by,

$$\rho_{ij}(t) = \langle i|\mathrm{Tr}_R U(t)\varrho(0)U^{-1}(t)|j\rangle, \quad i,j = 0,1,2,3, \tag{27}$$

where $\varrho(0) = \rho_s(0)\otimes|\Psi_V\rangle\langle\Psi_V|$. We start with calculation of the matrix element $\rho_{01}(t)$. Using (23) – (26) in (27), we obtain,

$$\rho_{01}(t) = e^{-2i\nu_{12}(\tau)}e^{-\mu(t)}\rho_{01}(0), \tag{28}$$

where

$$\mu = \frac{1}{2}\sum_{\alpha,\beta}\big(1+(-1)^{\alpha-\beta}\big)\mu_{\alpha\beta} - \xi_k^{1*}\xi_k^2. \tag{29}$$

Substituting $\xi_k^a = \kappa_a\xi_k$, into expression for μ, after some algebra we obtain,

$$\mu = 2\sum_{\mathbf{k}}|\xi_k(t)|^2 + 2i\sum_{\mathbf{k}}|\xi_k(t)|^2\sin(\mathbf{k}\cdot\mathbf{r}_{12}). \tag{30}$$

Now, taking into account all these relations, we get,

$$\rho_{01}(t) = e^{i\delta(t)}e^{-\gamma(t)}\rho_{01}(0), \tag{31}$$

where,

$$\delta(t) =-2\big(\nu_{12}(t) + \sum_{\mathbf{k}} \sin(\mathbf{k}\cdot\mathbf{r}_{12})|\xi_k(t)|^2\big), \tag{32}$$

$$\gamma(t) =2\sum_{\mathbf{k}} |\xi_k(t)|^2. \tag{33}$$

Similar computation of the other elements of the reduced density matrix yields our final result:

$$\rho_{ii}(t) =\rho_{ii}(0),\ i=0,1,2,3, \tag{34}$$

$$\rho_{01}(t) =e^{i\delta(t)}e^{-\gamma(t)}\rho_{02}(0), \tag{35}$$

$$\rho_{02}(t) =e^{i\delta(t)}e^{-\gamma(t)}\rho_{02}(0), \tag{36}$$

$$\rho_{03}(t) =e^{-\gamma_1(t)}\rho_{03}(0), \tag{37}$$

$$\rho_{12}(t) =e^{-\gamma_2(t)}\rho_{12}(0), \tag{38}$$

$$\rho_{13}(t) =e^{-i\delta(t)}e^{-\gamma(t)}\rho_{13}(0), \tag{39}$$

$$\rho_{23}(t) =e^{-i\delta(t)}e^{-\gamma(t)}\rho_{23}(0). \tag{40}$$

Here,

$$\delta(t) =-2\nu_{12}(t) + 2\sum_{\mathbf{k}} \sin(\mathbf{k}\cdot\mathbf{r}_{12})|\xi_k(t)|^2, \tag{41}$$

$$\gamma(t) =2\sum_{\mathbf{k}} |\xi_k(t)|^2, \tag{42}$$

$$\gamma_1(t) =8\sum_{\mathbf{k}} \cos^2\Big(\frac{\mathbf{k}\cdot\mathbf{r}_{12}}{2}\Big)|\xi_k(t)|^2, \tag{43}$$

$$\gamma_2(t) =8\sum_{\mathbf{k}} \sin^2\Big(\frac{\mathbf{k}\cdot\mathbf{r}_{12}}{2}\Big)|\xi_k(t)|^2. \tag{44}$$

The frequency shift

Using Eqs. (17) and (18), one can recast (32) as,

$$\delta(t) = -\frac{2\lambda^2}{V}\sum_{\mathbf{k}}\Big(\frac{k_z^2\cos(\mathbf{k}\cdot\mathbf{r}_{12})(\omega_k t-\sin(\omega_k t))}{\omega_k^3} + \frac{2k_z^2\sin(\mathbf{k}\cdot\mathbf{r}_{12})\sin^2(\omega_k t/2)}{\omega_k^3}\Big). \tag{45}$$

To proceed further, we replace a sum by an integral: $(1/V)\sum_{\mathbf{k}} \to (1/(2\pi)^3)\int d^3k$, and write the total phase as $\delta=\delta_1+\delta_2$, where,

$$\delta_1 = -\frac{\lambda^2}{4\pi^3}\int \frac{d^3k k_z^2\cos(\mathbf{k}\cdot\mathbf{r}_{12})(\omega_k t-\sin(\omega_k t))}{\omega_k^3}, \tag{46}$$

$$\delta_2 = -\frac{\lambda^2}{2\pi^3}\int \frac{d^3k k_z^2\sin(\mathbf{k}\cdot\mathbf{r}_{12})\sin^2(\omega_k t/2)}{\omega_k^3}. \tag{47}$$

We assume that spins are located at the points with the position vectors, $\mathbf{r}_1$ and $\mathbf{r}_2$. Without loss of generality, one can choose the orientation of the coordinate system in such a way that $\mathbf{r}_{12} = L(\sin\theta, 0, \cos\theta)$, where $L = |\mathbf{r}_{12}| \equiv r_{12}$. Using the spherical coordinates (k,ϑ,φ), one can recast Eqs. (46), (47) as,

$$\delta_1 = -\frac{\lambda^2}{2\pi^2}\int_0^\infty \frac{dk k^4 I_1(kL)(\omega_k t-\sin(\omega_k t))}{\omega_k^3}, \tag{48}$$

$$\delta_2 = -\frac{\lambda^2}{\pi^2}\int_0^\infty \frac{dk k^4 I_2(kL)\sin^2(\omega_k t/2)}{\omega_k^3}. \tag{49}$$

where,

$$I_1(kL) = \frac{1}{2\pi}\int_0^{2\pi} d\varphi\int_0^\pi d\vartheta \cos^2\vartheta\sin\vartheta\cos(\mathbf{k}\cdot\mathbf{r}_{12}),$$
$$I_2(kL) = \frac{1}{2\pi}\int_0^{2\pi} d\varphi\int_0^\pi d\vartheta \cos^2\vartheta\sin\vartheta\sin(\mathbf{k}\cdot\mathbf{r}_{12}). \tag{50}$$

The computation of the integrals $I_{1,2}$ yields (see Appendix B),

$$I_1(x) =\frac{2}{x^3}\big(x^2\sin x+3x\cos x-3\sin x\big)\cos^2\theta_0 + \frac{2}{x^3}\big(\sin x - x\cos x\big), \tag{51}$$

$$I_2(x) =0, \tag{52}$$

where a new dimensionless variable, $x=kL$, is introduced. Since $I_2=0$, we conclude that $\delta_2=0$.

Then, Eq. (45), determining the total phase, can be rewritten as,

$$\delta = -\frac{\lambda^2}{2\pi^2 L^3}\int_0^\infty \frac{dx x^4 I_1(x)(\omega t - L\sin(\omega t/L)}{\omega^3}, \tag{53}$$

where $\omega=\sqrt{x^2+(mL)^2}$.

In what follows, we assume that $mL \ll 1$. Then, one can recast (53) as,

$$\delta = \frac{\lambda^2 S_0(t)}{2\pi^2 L^3} + \mathcal{O}\big((mL)^2\big). \tag{54}$$

where

$$S_0(t) = \int_0^\infty dx x I_1(x)\big(L\sin(xt/L) - xt\big). \tag{55}$$

Performing the integration, we obtain (for detail see Appendix B),

$$\delta(t) = \begin{cases} 0, & \text{if } t<L, \\ \dfrac{\lambda^2 t}{\pi}\cdot\dfrac{(3\cos^2\theta-1)}{L^3}, & \text{if } t>L. \end{cases} \tag{56}$$

Decoherence

Here we limit ourselves by considering only the spin decoherence produced by the vacuum field. By replacing a sum by an integral, we have,

$$\gamma(t) = \frac{\lambda^2}{4\pi^3}\int \frac{d^3k k_z^2 \sin^2(\omega_k t/2)}{\omega_k^3}, \tag{57}$$

$$\gamma_1(t) = \frac{\lambda^2}{\pi^3}\int \frac{d^3k k_z^2 \cos^2\left(\frac{\mathbf{k}\cdot\mathbf{r}_{12}}{2}\right)\sin^2(\omega_k t/2)}{\omega_k^3}, \tag{58}$$

$$\gamma_2(t) = \frac{\lambda^2}{\pi^3}\int \frac{d^3k k_z^2 \sin^2\left(\frac{\mathbf{k}\cdot\mathbf{r}_{12}}{2}\right)\sin^2(\omega_k t/2)}{\omega_k^3}. \tag{59}$$

In what follows, we restrict ourselves by consideration only $\gamma(t)$. Performing integration over angle variables we obtain,

$$\gamma(t) = \frac{\lambda^2}{3\pi^2}\int_0^\infty \frac{dk k^4 \sin^2(\omega_k t/2)}{\omega_k^3}. \tag{60}$$

The asymptotic value of $\gamma(t)$, while $t \to \infty$, is given by,

$$\gamma(t) \to \gamma_0 = \frac{\lambda^2}{6\pi^2}\int_0^\infty \frac{dk k^4}{\omega_k^3}. \tag{61}$$

This integral is diverging as $k \to \infty$. There are different approaches to deal with this ultra-violet (UV) catastrophe, which occurs in many domains of the field theory. One of them is to introduce in (61) a cutoff, k_c. In this approach, the question on the value of k_c usually arises.

To regularize the divergent integral (61) we use the dimensional regularization, following the procedure described in [19]. For this purpose, consider the auxiliary integral,

$$\Sigma_\epsilon = \frac{2\pi^{\frac{3}{2}-\epsilon}}{\Gamma\left(\frac{3}{2}-\epsilon\right)}\int_0^\infty \frac{x^{4-2\epsilon}dx}{(x^2+1)^{3/2}}. \tag{62}$$

In order to calculate this integral we will use the formula:

$$\int_0^\infty \frac{r^\alpha dr}{(a+br^\beta)^\gamma} = \left(\frac{a}{b}\right)^{\frac{\alpha+1}{\beta}} \frac{\Gamma\left(\frac{\alpha+1}{\beta}\right)\Gamma\left(\gamma-\frac{\alpha+1}{\beta}\right)}{a^\gamma \beta\Gamma(\gamma)}. \tag{63}$$

The computation yields,

$$\Sigma_\epsilon = \frac{\pi^{\frac{3}{2}-\epsilon}\Gamma\left(\frac{5}{2}-\epsilon\right)\Gamma(\epsilon-1)}{\Gamma\left(\frac{3}{2}-\epsilon\right)\Gamma\left(\frac{3}{2}\right)}. \tag{64}$$

Using the identities $\Gamma(z+1) = z\Gamma(z)$ and $\pi^{-\epsilon} = e^{-\epsilon\ln\pi}$, we get

$$\Sigma_\epsilon = 2\pi e^{-\epsilon\ln\pi}\left(\frac{3}{2}-\epsilon\right)\Gamma(\epsilon-1). \tag{65}$$

For $\epsilon \ll 1$, we have $e^{-\epsilon\ln\pi} = 1-\epsilon\ln\pi+\mathcal{O}(\epsilon^2)$. To proceed further, we use the Laurent series expansion in a neighborhood of the pole $z=-1$:

$$\Gamma(z) = \Gamma(\epsilon-1) = -\frac{1}{\epsilon}+\gamma-1+\mathcal{O}(\epsilon), \tag{66}$$

where $\gamma = 0.57722$ is the Euler constant. After some algebra we obtain,

$$\Sigma_\epsilon = 3\pi\Big(-\frac{1}{\epsilon}+\gamma-\frac{1}{3}+\frac{2}{3}\ln\pi+\mathcal{O}(\epsilon)\Big). \tag{67}$$

We truncate the singular term $1/\epsilon$ of the function Σ_ϵ at the point $\epsilon = 0$ and obtain the regularized expression,

$$[\Sigma]_{reg} = \lim_{\epsilon\to 0}[\Sigma_\epsilon]_{reg} = \pi(3\gamma-1+2\ln\pi). \tag{68}$$

Returning to Eq.(46), we rewrite it as,

$$\gamma_0 = \frac{\lambda^2 m^2}{6\pi^2}\int_0^\infty \frac{x^4 dx}{(x^2+1)^{3/2}}, \tag{69}$$

where $x = k/m$. This can be recast as follows:

$$\gamma_0 = \frac{\lambda^2 m^2}{24\pi^3}\lim_{\epsilon\to 0}\Sigma_\epsilon. \tag{70}$$

Employing (68), we obtain,

$$\gamma_0 \to \gamma_0^{reg} = \frac{\lambda^2 m^2}{24\pi^2}(3\gamma-1+2\ln\pi). \tag{71}$$

Since for DM the product $\lambda m \ll 1$, we obtain $\gamma_0^{reg} \ll 1$. As a result, we have $e^{-\gamma_0^{reg}} \approx 1$. Thus, the partial phase decoherence takes place for $\rho_{01}(t)$. Similar conclusion can be made for other off-diagonal elements of the reduced spin density matrix. Note that the "dimensional regularization procedure", used here for decoherence rate, requires additional justification, which is not a subject of this paper.

Appendix B: Calculation of some useful integrals

Here we calculate the typical integrals emergent in the main text of the paper. We start with the integrals:

$$I_1 = \frac{1}{2\pi}\int_0^{2\pi} d\varphi \int_0^\pi d\vartheta \cos^2\vartheta \sin\vartheta \cos(\mathbf{k}\cdot\mathbf{r}_{12}), \tag{72}$$

$$I_2 = \frac{1}{2\pi}\int_0^{2\pi} d\varphi \int_0^\pi d\vartheta \cos^2\vartheta \sin\vartheta \sin(\mathbf{k}\cdot\mathbf{r}_{12}). \tag{73}$$

Here $\mathbf{k} = k(\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta)$. Without loss of generality, we can choose the orientation of the coordinate system in such a way that $\mathbf{r}_{12} = L(\sin\theta, 0, \cos\theta)$, where, as before, $L = |\mathbf{r}_{12}| \equiv r_{12}$. Before proceeding further, it is convenient to introduce new variables: $p = kL\sin\theta$ and $q = kL\cos\theta$. We obtain,

$$\mathbf{k}\cdot\mathbf{r}_{12} = p\sin\vartheta\cos\varphi + q\cos\vartheta. \tag{74}$$

First, consider the integral I_1 written as,

$$I_1 = \int_0^\pi f_1(\vartheta)\cos^2\vartheta\sin\vartheta d\vartheta, \tag{75}$$

where

$$f_1(\vartheta) = \frac{1}{2\pi}\int_0^{2\pi} \cos(p\sin\vartheta\cos\varphi + pq\cos\vartheta)d\varphi$$
$$= \cos(q\cos\vartheta)J_0(p\sin\vartheta). \quad (76)$$

Here $J_\nu(x)$ denotes the Bessel function. Introducing a new variable, $z = \sin\varphi$, one can recast (75) as,

$$I_1 = 2\Re\int_0^1 z\sqrt{1-z^2}e^{iq\sqrt{1-z^2}}J_0(pz)dz. \quad (77)$$

Performing the integration, we obtain [20]

$$I_1(kL) = -2\frac{\partial^2}{\partial q^2}\Big(\frac{\sin\sqrt{p^2+q^2}}{\sqrt{p^2+q^2}}\Big). \quad (78)$$

This yields

$$I_1(x) = \frac{2}{x^3}\big(x^2\sin x + 3x\cos x - 3\sin x\big)\cos^2\theta$$
$$+ \frac{2}{x^3}\big(\sin x - x\cos x\big). \quad (79)$$

Next, consider the integral I_2. Using (74), one can recast (73) as,

$$I_2 = 2\int_0^{\pi/2} f_2(\vartheta)\cos^2\vartheta\sin\vartheta\cos(q\cos\vartheta))d\vartheta, \quad (80)$$

where

$$f_2(\vartheta) = \frac{1}{2\pi}\int_0^{2\pi}\sin(p\sin\vartheta\cos\varphi)d\varphi. \quad (81)$$

As one can easily see, $f_2(\vartheta) = 0$, and thus the integral $I_2 = 0$.

Now, consider the integral,

$$S_0(t) = \int_0^\infty I_1(x)(L\sin(xt/L) - xt)xdx, \quad (82)$$

where $L > 0$. To evaluate this integral, we use the identities ($n \geq 0$):

$$\int_0^\infty \tau^n\cos(\kappa\tau)d\tau = \begin{cases} (-1)^{n/2}\pi\delta^{(n)}(\kappa), & n \quad \text{even} \\ (-1)^{(n+1)/2}\dfrac{n!}{\kappa^{n+1}}, & n \quad \text{odd} \end{cases} \quad (83)$$

$$\int_0^\infty \tau^n\sin(\kappa\tau)d\tau = \begin{cases} (-1)^{n/2}\dfrac{n!}{\kappa^{n+1}}, & n \quad \text{even} \\ (-1)^{(n+1)/2}\pi\delta^{(n)}(\kappa), & n \quad \text{odd} \end{cases} \quad (84)$$

where $\delta^{(n)}$ denotes the n^{th} derivative of the Dirac delta function.

Taking into account that,

$$\int_0^\infty \frac{\sin x}{x}dx = \frac{\pi}{2}, \quad (85)$$

after some algebra we obtain,

$$S_0(t) = \pi t(3\cos^2\theta - 1) + I_0, \quad (86)$$

where

$$I_0 = L\int_0^\infty I_1(x)\sin(xt/L)xdx. \quad (87)$$

The computation yields,

$$I_0 = L\int_0^\infty I_1(x)\sin(xt/L)xdx = \pi L\delta(t/L-1)$$
$$-\pi L\delta(t/L+1) + \frac{\pi t}{2}(3\cos^2\theta - 1)\big(H(t/L-1) - 1\big), \quad (88)$$

where $H(x)$ denotes the Heaviside function. Finally, we obtain

$$S_0(t) = \begin{cases} 0, & t < L \\ 2\pi t(3\cos^2\theta - 1), & t > L \end{cases} \quad (89)$$

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[1] Y.V. Stadnik and V.V. Flambaum, “Axion-induced effects in atoms, molecules, and nuclei: Parity nonconservation, anapole moments, electric dipole moments, and spin-gravity and spin-axion momentum couplings”, Phys. Rev. D, **89**, 043522 (2014).

[2] Y.V. Stadnik and V.V. Flambaum, “New generation low-energy probes for ultralight axion and scalar dark matter”, Mod. Phys. Lett. A, **32**, 1740004 (2017).

[3] R. Daido and F. Takahashi, “The sign of the dipole-dipole potential by axion exchange”, Phys. Lett. B, **772**, 127 (2017).

[4] Q. Shafi, S.H. Tanyildizi, and C.S. Un, “Neutralino dark matter and other LHC predictions from quasi Yukawa unification”, Nuclear Physics B, **900**, 400 (2015).

[5] K. Hamaguchi and K. Ishikawa, “Prospects for Higgs- and Z-resonant neutralino dark matter”, Phys. Rev. D, **93**, 055009 (2016).

[6] J. Bramante, N. Desai, P. Fox, A. Martin, B. Ostdiek, and T. Plehn, “Towards the final word on neutralino dark matter”, Phys. Rev D, **93**, 063525 (2016).

[7] N. Ypshimatsu, “Implications of heavy neutralino dark matter: The underlying structure in the hidden sector”, Int. J. Mod. Phys. A, **32**, 1750013 (2017).

[8] H. Fukuda, F. Luo, and S. Shirai, “How heavy can neutralino dark matter be?”, JHEP, **04**, 107 (2019).

[9] C.R. Argelles, A. Krut, J.A. Rueda, and R. Ruffini, “Novel constraints on fermionic dark matter from galactic observables I: The milky way”, Phys. Dark Univ., **21**, 82 (2018).

[10] C.R. Argelles, A. Krut, J.A. Rueda, and R. Ruffini, “Novel constraints on fermionic dark matter from galactic observables II: Galaxy scaling relations”, Phys. Dark Univ., **24**, 100278 (2019).
[11] R.T. Schumacher, *Introduction to Magnetic Resonance* (W.A. Benjumin, Inc, New York, 1970).
[12] E.W. Mielke and E.S. Romero, “Cosmological evolution of a torsion-induced quintaxion”, Phys. Rev. D, **73**, 043521 (2006).
[13] E. Calabrese and D.N. Spergel, “Ultra-light dark matter in ultra-faint dwarf galaxies”, MNRAS **460**, 43974402 (2016).
[14] G.P. Berman, V.N. Gorshkov, V.I. Tsifrinovich, M. Merkli, and X. Wang, “Bose Einstein condensate of ultra-light axions as a candidate for the dark matter galaxy halos”, Mod. Phys. Lett. A,**34**, 1950361 (2019).
[15] M. Goryachev, B.T. McAllister, and M.E. Tobar, “Axion detection with precision frequency metrology”, Physics of Dark Universe, **26**, 100345 (2019).
[16] M. Aaboud et al, “Search for charginoneutralino production using recursive jigsaw reconstruction in final states with two or three charged leptons in protonproton collisions at with the ATLAS detector, The ATLAS Collaboration”, Phys. Rev. D, **98**, 092012 (2018).
[17] G. Bertone and D. Hooper, “History of dark matter,” Rev. Mod. Phys. **90**, 045002 (2018).
[18] G.P. Berman, D.I. Kamenev, and V.I. Tsifrinovich, *Perturbation Theory for Solid-state Quantum Computation With Many Quantum Bits* (Rinton Pr Inc., 2005).
[19] E. Zeidler, *Quantum Field Theory II: Quantum Electrodynamics. A Bridge between Mathematicians and Physicists* (Springer-Verlag, Berlin Heidelberg, 2009).
[20] A.P. Prudnikov, Yu.A. Brychkov, and O.J. Marchev, *Integrals and Series: Volume 1: Elementary Functions; Volume 2: Special Functions* (CRC Press, 1986).