

Methods and Applications of Domain Decomposition for Fluid Flows

Hansong Tang, City College of New York, CUNY. 5/25/15

Objective This talk presents fundamentals as well as current status of DD for fluid flows, and its applications in fronts of various computational physics and engineering problems. The materials include classic work in DD for fluids as well as the author's work in this area in many years.

Content

I. Domain decomposition for hyperbolic conservation laws

As a theoretical foundation of governing equations for fluid flows, hyperbolic conservation laws and their discretization will be introduced. On this basis, DD methods to compute the conservation laws will be presented and analyzed with regard to their consistency, stability, and convergence. Typical algorithms at grid interfaces are presented, and one and higher dimensional situations will be discussed. Ref. e.g., [1--7]. The topics are

- An overview
- Interface algorithms
- Conservation error, stability, and weak solutions

II. DD for compressible flows

DD methods for compressible flows with backgrounds in aerospace engineering will be discussed, with emphasis on algorithms for treatments at grid interfaces. Difficulties including accurately and correctly capturing shock at grid interfaces will be illustrated and methods to overcome them will be presented. Ref. e.g., [4, 7-11]. The topics are

- Conservative and non-conservative Interface algorithms
- Interface treatments and non-physical solution

III. DD for incompressible flows

Methods for computing 3D, unsteady, incompressible flows are discussed, and several typical algorithms will be introduced, including a simple, effective algorithm proposed by the author for overset grids. Ref. [12--17]. The topics are

- Mass flux based-interpolation for grid interface
- Application to flows with complex geometry

IV. DD for environmental flows

Simulation of several important yet difficult environmental flow problems in actual engineering projects will be presented to illustrate the necessities and advantages of DD, which include stratified flows in reservoir and near fish intake, initial mixing of thermal effluent flows, etc. Ref. e.g., [14, 18].

V. DD for multiscale and multiphysics coastal ocean flows

Numerical simulation of coastal ocean flows has been greatly successful, but, strictly speaking, it is restricted to large-scale flows and individual phenomena. In recent years, various multiscale, multiphysics coastal ocean flows are emerging, such as Mexico Gulf oil spill and storm surge impinging and damaging bridges, and it has become urgent to develop our capability to simulate such flows. In view that there exist many mature models for individual phenomena at specific scales, coupling of these models via DD is the most promising and feasible approach for us to achieve such capability. In this talk, we will present our effort on integration of SIFOM, a model for small-scale, fully 3D, incompressible flows, and FVCOM, a model for estuary-scale coastal ocean flows via DD implemented with an overset grid method. The DD is not trivial; it involves coupling of different sets of PEDs, distinct algorithms, and dissimilar grids. Such effort is the first of its kind, and can deal with many multiphysics phenomena spanning a vast range of scales that are beyond the reach of other existing models. Numerical experiments on solution accuracy, convergence, etc. of the SIFOM-FVCOM system and its applications will be presented. Ref., e.g., [19--21].

VI. Challenges and open questions

A crucial issue in DD for fluids is grid interface treatment, and it has been studied intensively for incompressible flows in 1980s and 90s with regard to accuracy, stability, and convergence [5-11]. Now we are making efforts in integration of different flow models for coastal ocean flows (incompressible, with free surfaces) via DD [19-21], which involves coupling of different sets of PEDs, distinct algorithms, and dissimilar grids and is at fronts of DD and far more complicated than past DD problems. We will present involved open questions with regard to

- Model connection conditions leading to well posedness of the problems
- Model interface schemes that strictly assure certain properties such as conservation of mass

Progress on above issues will set up correct and solid foundations of above effort and produce seamless transition of solutions between different models and physical solutions.

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