Homework Assignment 4

MATH 2260 – Ordinary Differential Equations I

For Problems 1–2 below, determine the longest interval in which the given initial value problem is certain to have a unique twice differentiable solution. Do not attempt to find the solution.

1.

$$(t-1)y'' - 3ty' + 4y = \sin t, \qquad y(-2) = 2, \qquad y'(-2) = 1.$$

2.

$$(x-2)y'' + y' + (x-2)(\tan x)y = 0, \qquad y(3) = 1, \qquad y'(3) = 2.$$

3. Find a fundamental set of solutions for the differential equation and initial point

$$y'' + y' - 2y = 0, \qquad t_0 = 0.$$

4. Consider the equation

$$y'' - y' - 2y = 0.$$

(a) Show that

$$y_1(t) = e^{-t}, \qquad y_2(t) = e^{2t}$$

form a fundamental set of solutions.

(b) Let

$$y_3(t) = -2e^{2t}, \qquad y_4(t) = y_1(t) + 2y_2(t), \qquad y_5(t) = 2y_1(t) - 2y_3(t).$$

Are $y_3(t)$, $y_4(t)$, and $y_5(t)$ also solutions of the given differential equation?

(c) Determine whether each of the following pairs forms a fundamental set of solutions:

$$[y_1(t), y_3(t)], \qquad [y_2(t), y_3(t)], \qquad [y_1(t), y_4(t)], \qquad [y_4(t), y_5(t)].$$

5. Find the Wronskian of two solutions of the given differential equation without solving the equation:

$$(\cos t)y'' + (\sin t)y' - ty = 0.$$

For Problem 6 below, find the general solution of the given differential equation.

6.

$$y'' + 6y' + 13y = 0.$$

For Problem 7 below, find the solution of the given initial value problem. Sketch the graph of the solution and describe its behavior for increasing t.

7.

$$y'' + 4y' + 5y = 0, \qquad y(0) = 1, \qquad y'(0) = 0.$$

8. Let the real-valued functions p and q be continuous on the open interval I, and let

$$y = \phi(t) = u(t) + iv(t)$$

be a complex-valued solution of

$$y'' + p(t)y' + q(t)y = 0,$$

where u and v are real-valued functions. Show that u and v are also solutions of this differential equation.

Hint: Substitute $y = \phi(t)$ in the differential equation and separate into real and imaginary parts.