

Homework Assignment 3

MATH 2260 – Ordinary Differential Equations I

For Problems 1–2 below, sketch the graph of $f(y)$ versus y , determine the critical (equilibrium) points, and classify each one as asymptotically stable or unstable. Draw the phase line, and sketch several graphs of solutions in the ty -plane.

1.

$$\frac{dy}{dt} = ay + by^2, \quad a > 0, \quad b > 0, \quad y_0 \geq 0.$$

2.

$$\frac{dy}{dt} = y(y-1)(y-2), \quad y_0 \geq 0.$$

3. Suppose that a certain population obeys the logistic equation

$$\frac{dy}{dt} = ry \left(1 - \frac{y}{K}\right).$$

(a) If $y_0 = K/3$, find the time τ at which the initial population has doubled. Find the value of τ corresponding to $r = 0.025$ per year.

(b) If $y_0/K = \alpha$, find the time T at which $y(T)/K = \beta$, where $0 < \alpha, \beta < 1$. Observe that $T \rightarrow \infty$ as $\alpha \rightarrow 0$ or as $\beta \rightarrow 1$. Find the value of T for $r = 0.025$ per year, $\alpha = 0.1$, and $\beta = 0.9$.

For Problems 4–5 below, determine whether each equation is exact. If it is exact, find the solution.

4.

$$(2x + 4y) + (2x - 2y)y' = 0.$$

5.

$$(2xy^2 + 2y) + (2x^2y + 2x)y' = 0.$$

For Problems 6–7 below, find an integrating factor and solve the given equation.

6.

$$y \, dx + (2xy - e^{-2y}) \, dy = 0.$$

7.

$$\left(\frac{4x^3}{y^2} + \frac{3}{y}\right) dx + \left(\frac{3x}{y^2} + 4y\right) dy = 0.$$

For Problems 8–9 below, find the general solution of the given differential equation.

8.

$$y'' + 3y' + 2y = 0.$$

9.

$$y'' - 2y' - 2y = 0.$$

10. Find the solution of the initial value problem

$$2y'' - 3y' + y = 0, \quad y(0) = 2, \quad y'(0) = \frac{1}{2}.$$

Then determine the maximum value of the solution and also find the point where the solution is zero.