

Homework Assignment 2

MATH 2260 – Ordinary Differential Equations I

For Problems 1–3 below, solve the given differential equation.

1.

$$\frac{dy}{dx} = \frac{x^2}{y(1+x^3)}.$$

2.

$$\frac{dy}{dx} = \cos^2 x \cos^2(2y).$$

3.

$$\frac{dy}{dx} = \frac{x - e^{-x}}{y + e^y}.$$

4. A tank initially contains 120 L of pure water. A mixture containing a concentration of γ g/L of salt enters the tank at a rate of 2 L/min, and the well-stirred mixture leaves the tank at the same rate. Find an expression in terms of γ for the amount of salt in the tank at any time t . Also find the limiting amount of salt in the tank as $t \rightarrow \infty$.

For Problem 5 below, determine (without solving the problem) an interval in which the solution of the given initial value problem is certain to exist.

5.

$$t(t-4)y' + y = 0, \quad y(2) = 1.$$

For Problem 6 below, state where in the ty -plane the hypotheses of Theorem 2.4.2 are satisfied.

6.

$$y' = \sqrt{1 - t^2 - y^2}.$$

7. (a) Show that $\phi(t) = e^{2t}$ is a solution of $y' - 2y = 0$ and that $y = c\phi(t)$ is also a solution of this equation for any value of the constant c .
(b) Show that $\phi(t) = 1/t$ is a solution of $y' + y^2 = 0$ for $t > 0$ but that $y = c\phi(t)$ is not a solution of this equation unless $c = 0$ or $c = 1$. Note that the equation in part (b) is nonlinear, while that in part (a) is linear.

8. Let $y = y_1(t)$ be a solution of

$$y' + p(t)y = 0,$$

and let $y = y_2(t)$ be a solution of

$$y' + p(t)y = g(t).$$

Show that $y = y_1(t) + y_2(t)$ is also a solution of the second equation.