

Homework Assignment 1

MATH 2260 – Ordinary Differential Equations I

1. Consider the differential equation $\frac{dy}{dt} = ay - b$, where $a \neq 0$ and b are two real numbers.
 - (a) Find the equilibrium solution y_e .
 - (b) Let $Y(t) = y - y_e$; thus $Y(t)$ is the deviation from the equilibrium solution. Find the differential equation satisfied by $Y(t)$.
2. The field mouse population satisfies the differential equation $\frac{dp}{dt} = 0.5p - 450$.
 - (a) Find the time at which the population becomes extinct if $p(0) = 850$.
 - (b) Find the time of extinction if $p(0) = p_0$, where $0 < p_0 < 900$.
 - (c) Find the initial population p_0 if the population is to become extinct in 1 year.
3. The falling object satisfies the initial value problem

$$\frac{dv}{dt} = 9.8 - \frac{v}{5}, \quad v(0) = 0.$$

- (a) Find the time that must elapse for the object to reach 98% of its limiting velocity.
- (b) How far does the object fall in the time found in part (a)?

For Problems 4 and 5 below, determine the order of the differential equation and state whether the equation is linear or nonlinear.

4.

$$t^2 \frac{d^2 y}{dt^2} + t \frac{dy}{dt} + 2y = \sin t.$$

5.

$$\frac{dy}{dt} + ty^2 = 0.$$

6. Determine the values of r for which the differential equation $y'' + y' - 6y = 0$ has a solution of the form $y = e^{rt}$.

For Problems 7–9 below, find the solution of the given initial value problem.

7.

$$y' - y = 2te^{2t}, \quad y(0) = 1.$$

8.

$$y' + \frac{2}{t}y = \frac{\cos t}{t^2}, \quad y(\pi) = 0, \quad t > 0.$$

9.

$$t^3 y' + 4t^2 y = e^{-t}, \quad y(-1) = 0, \quad t < 0.$$