Assignment 3, MATH 6102

1. For the multi-type SIS epidemic model, i.e., the system of ODEs on $(S(t), I(t))$ with $S(t) = (S_1(t), \ldots, S_n(t))$ and $I(t) = (I_1(t), \ldots, I_n(t))$, write down the basic reproduction number R_0, and then establish a threshold dynamics result in terms of R_0. Note that you can use the known result on the proportion $y(t) = (y_1(t), \ldots, y_n(t))$, where $y_i(t) = \frac{I_i(t)}{\sigma_i}$, $1 \le i \le n$, and σ_i is the total population of type i.

2. Consider the following criss-cross venereal infection model with the removed class permanently immune:

$$\frac{dS}{dt} = -rSI', \quad \frac{dS'}{dt} = -r'S'I,$$
$$\frac{dI}{dt} = rSI' - aI, \quad \frac{dI'}{dt} = r'S'I - a'I',$$
$$\frac{dR}{dt} = aI, \quad \frac{dR'}{dt} = a'I',$$

where the parameters are all positive. The initial values for S, I, R, S', I' and R' are $S_0, I_0, 0, S'_0, I'_0$ and 0, respectively. Show that the threshold condition for an epidemic to occur is at least one of

$$\frac{S_0 I'_0}{I_0} > \frac{a}{r}, \quad \frac{S'_0 I_0}{I'_0} > \frac{a'}{r'}.$$