Assignment 2, MATH 6104

1. Let (E, P) be an ordered Banach space with the positive cone P having nonempty interior. Prove that if $\lim_{n\to\infty} u_n = u$ and $u \ll u^*$ in E, then there exists some $n_0 > 0$ such that $u_n \ll u^*$, $\forall n \geq n_0$.

2. First write down the definition of the irreducibility for a square matrix. Then obtain a set of sufficient conditions for an ODE system $u' = F(u), u \in \mathbb{R}^n$ to generate a strongly monotone semiflow on $\mathbb{R}^n$.

3. Let (E, P) be an ordered Banach space with the positive cone P having nonempty interior, and let either $V = [0, b]$ with $b \gg 0$ in E, or $V = P$. Assume that $S : V \to V$ is continuous, $S(0) = 0$ and $DS(0)$ exists. Prove that if S is subhomogenous, then $S(u) \leq DS(0)u, \forall u \in V$, and that if S is strictly subhomogenous, then $S(u) < DS(0)u, \forall u \in V \cap Int(P)$.

4. Use the monotone semiflow theory to establish a threshold type result on the global dynamics of the following SIS epidemic model defined on $W = [0, 1]^n$:

$$\frac{dy_i(t)}{dt} = (1 - y_i(t)) \sum_{j=1}^{n} \sigma_j \lambda_{ij} y_j(t) - \mu_i y_i(t), \quad 1 \leq i \leq n.$$

Here all constants are positive real numbers and the matrix $\Lambda := (\sigma_j \lambda_{ij})_{n\times n}$ is irreducible.

5. Prove that the existence, uniqueness and Liapunov stability of T-periodic solutions of a T-periodic ODE system $u' = f(t, u) \equiv f(t + T, u)$ are equivalent to those of the fixed points of its Poinaré (period) map. Note that you may impose some reasonable conditions on the given perioldic system, if necessary.