

Assignment 2, MATH 6102

1. The interaction between two populations with densities N_1 and N_2 is modeled by

$$\begin{aligned}\frac{dN_1}{dt} &= rN_1 \left(1 - \frac{N_1}{K}\right) - aN_1N_2(1 - e^{-bN_1}), \\ \frac{dN_2}{dt} &= -dN_2 + eN_2(1 - e^{-bN_1}),\end{aligned}\tag{1}$$

where a, b, d, e, r and K are positive constants. What type of interaction exists between N_1 and N_2 ? What do the various terms in (1) imply ecologically?

2. Assume that two species interacting system

$$\begin{aligned}\frac{du_1}{dt} &= u_1G_1(u_1) \\ \frac{du_2}{dt} &= u_2G_2(u_1, u_2)\end{aligned}\tag{2}$$

satisfies the following conditions:

(C1) $\frac{dG_1}{du_1} < 0$, $\forall u_1 \in \mathbb{R}_+$, and $G_1(K_1) \leq 0$ for some $K_1 > 0$;

(C2) $\frac{\partial G_2}{\partial u_1} \geq 0$ and $\frac{\partial G_2}{\partial u_2} < 0$, $\forall (u_1, u_2) \in \mathbb{R}_+^2$, and for each $u_1 > 0$, there exists $K_2 = K_2(u_1) > 0$ such that $G_2(u_1, K_2) \leq 0$.

Find a set of sufficient conditions under which system (2) has a globally attractive positive equilibrium in $Int(\mathbb{R}_+^2)$.

3. Consider the competitor-competitor-mutualist system

$$\begin{aligned}\frac{du_1}{dt} &= \beta_1 u_1 \left[1 - \frac{u_1}{a_1} - \frac{a_2 u_2}{1 + a_3 u_3}\right], \\ \frac{du_2}{dt} &= \beta_2 u_2 \left[1 - b_1 u_1 - \frac{u_2}{b_2}\right], \\ \frac{du_3}{dt} &= \beta_3 u_3 \left[1 - \frac{u_3}{c_0 + c_1 u_1}\right],\end{aligned}\tag{3}$$

where all constants β_i, a_i, b_i and c_i are positive. Use the persistence theory to find a set of conditions under which three species are uniformly persistent and system (3) admits at least one positive equilibrium.