An International Workshop on
Nonlinear Dynamical Systems with Applications

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Organizers: Hermann Brunner and Xiaoqiang Zhao, Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, NF A1C 5S7, Canada. E-mail: hermann@math.mun.ca xzhao@math.mun.ca

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Purpose: This workshop will provide an opportunity for scientists in the fields of differential equations and dynamical systems with applications to communicate current research results, ideas and problems, to discuss future research directions, and to initiate research collaborations in the conducive small setting of an Oberwolfach/ BIRS-type workshop.

Topics:

1. Asymptotic behavior in finite and infinite dimensional dynamical systems;
2. Special solutions in partial differential equations(steady states, waves etc);
3. Hamiltonian systems and topological dynamics;
4. Bifurcations and chaos in physics and biology;
5. Evolution equations and population dynamics;
6. Numerical analysis and differential equations.

Abstracts:

Hermann Brunner, Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, NF A1C 5S7, Canada. Email: hermann@math.mun.ca

Collocation Methods for Functional Integro-Differential Equations with Proportional Delays

While the global convergence properties of piecewise polynomial collocation solutions to Volterra functional integro-differential equations of the form

$$y^{(k)}(t) = a(t)y(t) + b(t)y(qt) + (V_q y)(t), \quad t \in I := [0,T] \ \ (k \geq 1), \qquad (1)$$

with

$$(V_q y)(t) := \int_{qt}^{t} K(t,s)y(s)ds, \quad 0 < q < 1,$$

are now well understood (e.g. [2]), the analysis of optimal superconvergence and asymptotic stability for uniform meshes remains largely open. In this talk I shall survey recent work on local and global superconvergence results and numerical stability for certain types of geometric meshes ([3, 1]) and then describe current approaches for dealing with the analogous analysis on uniform meshes. A discussion of open problems, e.g. the convergence of collocation solutions for $(V_q y)(t) = g(t)$ (which may for example occur as part of a system of "integral-algebraic" equations), will complement this presentation.

References

[1] A. Bellen, Preservation of superconvergence in the numerical integration of delay differential equations with proportional delay, *IMA J. Numer. Anal.*, to appear.

[2] H. Brunner, *Collocation Methods for Volterra Integral and Related Functional Differential Equations*, Cambridge University Press, to appear.

[3] H. Brunner, Q. Hu and Q. Lin, Geometric meshes in collocation methods for Volterra integral equations with proportional delays, *IMA J. Numer. Anal.*, 21 (2001), 783-798.

Teresa Faria, Department of Mathematics and CMAF, University of Lisbon, 1749-016 Lisboa, Portugal. E-mail: tfaria@lmc.fc.ul.pt

Global and Asymptotic Behaviour for Some Delayed Scalar Population Models

In this talk, we first consider a delayed logistic equation with an autonomous functional part in the form

$$\dot{x}(t) = b(t)x(t)[1 - L(x_t)], \tag{1}$$

where $L : C([-r,0], R) \to R$ is a positive linear operator and b a positive continuous function. We suppose that the operator L carries some weight at zero, but relax the usual requirement that L has an undelayed component that dominates the delayed part. Sufficient conditions for the global stability of the positive equilibrium x_* are established. We also prove that x_* is the only solution on R that is bounded and bounded below from zero. This latter result, although it seems irrelevant in biological terms, is used to address the global asymptotic stability of more general delayed logistic models,

$$\dot{x}(t) = b(t)x(t)[a(t) - L(t, x_t)], \tag{2}$$

with $L(t, \cdot)$ linear and positive. This equation is assumed to be a perturbation of (1), in the sense that (2) becomes (1) in the limit $t \to \infty$. The techniques developed for the situation of L positive can be applied to the case of Eq. (1) with L a non-positive linearity L. We establish conditions that guarantee that positive solutions are defined and bounded on $[0, \infty)$, as well as that the positive equilibrium (if it exists) is globally stable. This a joint work with Eduardo Liz (University of Vigo, Spain) and Clotilde Martínez (University of Granada,Spain).

Andy Foster, Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, NF A1C 5S7, Canada. E-mail: foster@math.mun.ca

Bifurcating Structures in a Nonlinear Oscillator with Strong Periodic Forcing

The behavior resulting from imposition of seasonal effort harvesting on a species in a continuous predator-prey model (Rosenzweig-MacArthur) is studied. Large amplitude periodic harvesting in such biological population models typically results in richly diverse and complicated bifurcation structures. Exhaustive analysis of these structures is impossible at present, but they can be understood through numerical studies and classification of generic bifurcation sequences. High-order return maps are utilized to interpret mechanisms for some of the characteristic flow behaviors, including distinctive phase locking regions, chaotic transients and local chaotic attractors. These return maps are constructed similarly to Poincaré maps, and they can reduce the essential flow behavior to the behavior of noninvertible scalar maps.

Yuxia Guo, Department of Mathematics, Tsinghua University, Beijing 100084, China. E-mail: yguo@math.tsinghua.edu.cn

Morse Theory for Strongly Indefinite Functional

In this talk, a new Morse theory is constructed for a class of strongly indefinite functional of the form $f(x) = \frac{1}{2}\langle Ax, x\rangle + G(x)$, where A is a linear bounded selfadjoint operator and ∇G is compact; Generalized Morse inequalities for Morse decompositions of a dynamically isolated invariant sets are given.

Mats Gyllenberg, Department of Mathematics, University of Turku, Finland. E-mail: matsgyl@mailhost.utu.fi

Formulation and Analysisis of Structured Population Models

John R. Haddock, Office of Academic Affairs, The University of Memphis, Memphis, Tennessee 38152, USA. E-mail: jhaddock@memphis.edu

Some Principles in Dynamical Systems

Conditions are developed with respect to positive valued functions along forward flows to establish principles in dynamical systems related to repellers, instability relative to certain sets, and persistence. Examples to illustrate the results are provided from finite and infinite delay functional differential equations (discrete and distributed delay).

Stuart Hastings, Department of Mathematics, University of Pittsburgh, Pittsburgh, PA 15260, USA. E-mail: sph+@pitt.edu

Spikes, Layers and Chaos in a Spatially Inhomogeneous
Reaction Diffusion Equation

We consider the reaction-diffusion equation $u_t = \varepsilon^2 u_{xx} + f(u,x)$ where f has appropriate cubic-like behavior in u. With Neumann boundary conditions we look for steady-state solutions and find new types including both spikes and multiple transition layers. The ODE satisfied by steady states can also be studied on an infinite interval, and then we find a weak kind of chaos, similar to that of a "topological horseshoe" map. Such behavior is found for a specific range of ε, rather than just "small" ε. If ε is sufficently small, then we obtain a stronger result, with sensitivity to initial conditions equivalent to the usual hyperbolic horseshoe. This is joint work with S. Ai and with Ai and X. Chen.

Jingtang Ma, Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, NF A1C 5S7, Canada. E-mail: jingtang@math.mun.ca

Discontinuous Galerkin Method for Ordinary Differential Equations and
Volterra Integro-Differential Equations

Discontinuous Galerkin method for ordinary differential equation and (semi-linear) Volterra Integro-differential equation is discussed. Both a priori and a posteriori error estimates are given. Furthermore, Contribution of quadrature of the memory term to the total error is also studied. At last the comparison with classical Galerkin method and collocation method is analysed through numerical examples.

Angelo Mingarelli, School of Mathematics and Statistics Carleton University, Ottawa, ON K1S 5B6, Canada. E-mail: amingare@math.carleton.ca

Oscillation of Linear Hamiltonian Systems

We establish new oscillation criteria for linear Hamiltonian systems using monotone functionals on a suitable matrix space. In so doing we develop new criteria for oscillation involving general monotone functionals instead of the usual largest eigenvalue. Our results are new even in the particular case of self-adjoint second order differential systems. (Joint with Fanwei Meng, Proc. Amer. Math. Soc., in press)

James Muldowney, Dept of Mathematical and Statistical Sciences, University of Alberta, Edmonton, AB T6G 2G1, Canada. E-mail: jim.muldowney@ualberta.ca

Implications of The Stability of Linarizations

Let $x = \phi(t)$ be a bounded solution of the C^1 autonomous system $\dot{x} = f(x)$ in R^n. It is an exercise to show that the omega limit set of this solution is a stable hyperbolic equilibrium if and only if the linearized system $\dot{y} = \frac{\partial f}{\partial x}(\phi(t))\, y$ is uniformly asymptotically stable. This talk will present similar conditions for the omega limit set to be a stable hyperbolic periodic orbit or a homoclinic or heteroclinic cycle with certain attraction properties. Work is joint with Michael Li.

George Sell, School of Mathematics, University of Minnesota, 206 Church Street SE, Minneapolis, MN 55455, USA. E-mail: sell@math.umn.edu

Predictions of the El Nino Event: A Mathematical Perspective

Horst Thieme, Department of Mathematics,Arizona State University, Tempe, AZ 85287-1804, USA. E-mail: thieme@math.la.asu.edu

Traveling Waves and Asymptotic Speed of Spread for Integral Equations Revisited in The Light of Diffusion Equations With Delay

Dashun Xu, Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, NF A1C 5S7, Canada. E-mail: dxu@math.mun.ca

Stability and Hopf Bifurcation in a Structured Population Model

In this talk, we consider a system of delay differential equations which describes a structured population on two patches. For a large class of birth functions, we get sufficient conditions for uniform persistence and global stabilities of equilibria . A Hopf bifurcation in the system is also discussed when the birth function takes a specific form, and the stability of the bifurcated periodic solutions and the bifurcation direction are investigated in details. Finally,some numerical simulations of the system are given to illustrate the Hopf bifurcation.

Jianhong Wu, Department of Mathematics and Statistics, York University North York, Ontario M3J 1P3, Canada. E-mail: wujh@mathstat.yorku.ca

Non-Local Interaction through Spatial Diffusion and Temporal Delay: Dynamics and Biological Applications

We report some recent progress towards modeling and dynamics analysis as well as numerical simulations for a new class of reaction-diffusion equations with non-local interaction, when both time delay and spatial diffusion are simultaneously involved.

Yingfei Yi, School of Mathematics, Georgia Institute of Technology, Atlanta, GA, USA. E-mail: yi@math.gatech.edu

Traveling Wave Solutions in a Tissue Interaction Model

We discuss the existence and the uniqueness of traveling wave solutions for a tissue interaction model for skin pattern formation proposed by Cruywagen and Murray. By employing the geometric singular perturbation theory with two time scales, insight of the geometric structure of the phase space for traveling wave solutions will be explored.