PREFACE
Introduction

1. Introduction

The study of traveling waves and spreading speeds is an important area of research in mathematical biology. These concepts have been extensively studied in the context of reaction-diffusion equations, which model the spread of an organism or an idea through a population.

Spreading speeds play a crucial role in understanding the invasion dynamics of species. The spreading speed is defined as the minimal speed at which a front of invasion of the species propagates through the environment. This concept is closely related to the concept of traveling waves, which are solutions of the reaction-diffusion equation that maintain their shape while propagating at a constant speed.

The following theorems provide a framework for understanding the behavior of solutions to reaction-diffusion equations in the context of spreading speeds.

Theorem A. Let \( u(x,t) \) be a nontrivial solution of \( (1) \) with \( u(0) \in V \). Then the following two statements are equivalent:

1. \( u(x,t) \) has a spreading speed \( c \).
2. \( \lim_{t \to \infty} u(x,t) = \phi(x) \) for some function \( \phi \).

Theorem B. Let \( u(x,t) \) be a nontrivial solution of \( (1) \) with \( u(0) \in V \). Then the following two statements are equivalent:

1. \( u(x,t) \) has a spreading speed \( c \).
2. \( \lim_{t \to \infty} u(x,t) = \phi(x) \) for some function \( \phi \) that is positive and bounded.

These theorems provide a rigorous mathematical framework for understanding the spread of invasive populations and the role of spreading speeds in determining the rate of invasion.
2.1. Monotone case

Spaces of functions

The spaces for order-preserving (monotone) maps and semigroups on ordered sets are of interest in this section. In this section, we discuss spreading speeds and traveling wave speeds for monotone and non-monotone systems. Non-monotone systems were the subject of the theory and methods presented in papers with collaborations.

Theorem B. Equation (1) admits a unique (up to translation) monotone non-decreasing solution $u^*$ for the following initial conditions:

$$u(x, 0) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$$

where $\epsilon > 0$.

Proof. By the comparison principle, the solution $u^*$ is unique and non-decreasing. The existence of a solution can be shown by constructing a sequence of functions that converge uniformly to $u^*$.

Example: Consider the reaction-diffusion equation

$$u_t = D u_{xx} + f(u),$$

with $f(u) = u^p - u$, where $p > 1$ and $D > 0$. The solution $u^*$ is a traveling wave with a constant speed $c^*$. The speed $c^*$ is given by

$$c^* = \frac{1}{2} \sqrt{f'(1)}.$$
If $\mathcal{H} \in C^1([0,1])$, then $\mathcal{H}$ is a positive operator. Let $\mathcal{H}$ be a positive operator. Assume that

$$\int_0^1 \int_0^1 |\mathcal{H}(x'\theta)[n]\mathcal{H}^* (x'\theta)[n] dx\,d\theta = \int_0^1 \int_0^1 |\mathcal{H}(x'\theta)[n]\mathcal{H}^* (x'\theta)[n] dx\,d\theta$$

For any $\mathcal{H} \in C^1([0,1])$, the linear operator $\mathcal{H}$ is bounded by

$$\int_0^1 \int_0^1 \lambda \mathcal{H}(x'\theta)[n] \mathcal{H}^* (x'\theta)[n] dx\,d\theta \leq \int_0^1 \int_0^1 \lambda \mathcal{H}(x'\theta)[n] \mathcal{H}^* (x'\theta)[n] dx\,d\theta$$

Since $\mathcal{H} \in C^1([0,1])$, it follows that $\mathcal{H}$ is bounded.

Theorem 2.2: Let $\mathcal{H}$ be an operator on $C^1([0,1])$, then $\mathcal{H}$ is a positive operator. Assume that

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Since $\mathcal{H} \in C^1([0,1])$, it follows that $\mathcal{H}$ is bounded.
Theorem 2.2: There exists an integer \( m \) such that for all \( n \geq m \), the inequality \( \left| \frac{1}{n} \right| < \epsilon \) holds. Assume that \( \epsilon > 0 \) and let \( (f(n))_{n=1}^{\infty} \) be a sequence of real numbers. Then, for any \( \epsilon > 0 \), there exists a positive integer \( N \) such that for all \( n \geq N \), the inequality \( \left| f(n) - L \right| < \epsilon \) holds, where \( L \) is the limit of \( (f(n))_{n=1}^{\infty} \).

Remark 2.1: Theorem 2.2 is a direct consequence of the completeness of the real numbers.

The theory of reaction-diffusion equations is the study of how concentrations of chemical species change over time. In this context, the reaction term represents the production or consumption of the chemical, while the diffusion term describes how the chemical spreads out due to random motion.

Note that for a general reaction-diffusion equation of the form \( \frac{\partial u}{\partial t} = \Delta u + f(u) \), where \( u \) is the concentration of the chemical and \( f(u) \) is a function of the concentration, the behavior of solutions depends on the properties of \( f(u) \) and the initial conditions.

We shall prove that under certain conditions, the solutions to reaction-diffusion equations converge to a steady state as time goes to infinity. This is a fundamental result in the study of reaction-diffusion systems and has applications in various fields such as biology, chemistry, and physics.
Assume that

\[
(n^2 + 2n + 1) = \frac{10}{n^2}
\]

Let \( n^2 \neq x \).

Consider a reaction-diffusion system

\[
\tag{2.3}
\]

which represents the steady state obtained in the limit. In this case, the system is given by

\[
\tag{2.4}
\]

The approach was used to prove the global attractivity of the system.

Theorem 2.2: For any positive continuous function \( f(x) \), there exists a unique solution \( x(t) \) of the following system:

\[
\tag{2.5}
\]

where \( f(x) \) is a positive continuous function and \( g(x) \) is a continuously differentiable function.

2.2.1 Duality case

The condition that \( 0 < (1) \),\( f + (1)^2 \neq 0 \), and \( (1)^2 \neq 0 \), is not strongly monotone.

For any interval \( I \), the quadratic term

\[
\tag{2.6}
\]

is strongly monotone.

Theorem 2.2: For any interval \( I \), the quadratic term

\[
\tag{2.7}
\]

is strongly monotone.

Theorem 2.2: For any interval \( I \), the quadratic term

\[
\tag{2.8}
\]

is strongly monotone.

Theorem 2.2: For any interval \( I \), the quadratic term

\[
\tag{2.9}
\]

is strongly monotone.
Assume that the kernel \( k(x, y) \) has the following property:

\[
\langle f, g \rangle = \int \int f(x)k(x, y)g(y)dx\,dy.
\]

Let be a non-negative, increasing measurable function on \( \mathbb{R}^d \).

Then, for any \( \varepsilon > 0 \), there exists a number \( T > 0 \) such that

\[
\int_{|x|>T} k(x, y)dx < \varepsilon.
\]

Let \( \mathcal{F} \) be the space of all bounded and continuous functions from \( \mathbb{R}^d \) to \( \mathbb{C} \). Let \( \mathcal{F} \) be the Banach space of all bounded and continuous functions from \( \mathbb{R}^d \) to \( \mathbb{C} \). Let \( \mathcal{F} \) be the Banach space of all bounded and continuous functions from \( \mathbb{R}^d \) to \( \mathbb{C} \).

Then \( \mathcal{F} \) is a Banach space with the usual supremum norm.

\[
\|f\|_{\mathcal{F}} = \sup_{x \in \mathbb{R}^d} |f(x)|.
\]

because the Banach space of all bounded and continuous functions from \( \mathbb{R}^d \) to \( \mathbb{C} \) is complete.

\[
\|f\|_{\mathcal{F}} = \sup_{x \in \mathbb{R}^d} |f(x)|.
\]

Theorem 2.10 Let \( \mathcal{F} \) be a Banach space of all bounded and continuous functions from \( \mathbb{R}^d \) to \( \mathbb{C} \).

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Then \( \mathcal{F} \) is a Banach space with the usual supremum norm.
Consider a discrete-time linear difference equation
\[
\sum_{k=0}^{n-1} \alpha_k y(x-k) = f(x)
\]
with \( n \geq 1 \). Assume that there exists \( 0 < \theta < b \) such that
\[
\int_{\theta}^{b} \left( x \right)^{n-1} dx = \int_{\theta}^{b} (x)^{1+n} dx
\]
and
\[
\alpha_k \in \mathbb{C}, \quad 0 \leq k \leq n-1.
\]

Now we consider the discrete-time linear difference equation
\[
\sum_{k=0}^{n-1} \alpha_k y(x-k) = f(x)
\]
with \( n \geq 1 \). Assume that there exists \( 0 < \theta < b \) such that
\[
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\]
and
\[
\alpha_k \in \mathbb{C}, \quad 0 \leq k \leq n-1.
\]

The following results were proved in [25] under the assumptions that

1. \( f(x) \) is a \( C^n \) function,
2. \( \alpha_k \) are complex constants,
3. \( n \geq 1 \), and
4. \( \theta, b \) are real numbers.

Theorem 3.2. Let \( f(x) \) be a \( C^n \) function. Then

\[
\int_{\theta}^{b} \left( x \right)^{n-1} dx = \int_{\theta}^{b} (x)^{1+n} dx
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holds for any \( \theta, b \) with \( 0 < \theta < b \). The proof of the theorem is similar to the proof of Theorem 3.1.

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where we have the following implication

\[ \forall \alpha > 0, \exists \delta > 0 \text{ s.t. } \forall \epsilon > 0, \exists N > 0 \text{ s.t. } \forall n > N, |x_n - \alpha| < \epsilon. \]

By the theory of non-uniformly hyperbolic systems, we have

**Theorem 4.1.2** \text{THEOREMS 2.1 AND 2.2.} Assume that \((\Omega, \mathcal{F}, \mu, T)\) is a measure-preserving transformation, and let \(f \in C^1(\Omega, \mathbb{R})\) be a \(C^1\)-smooth function. If for every \(x \in \Omega\), there exists a neighborhood \(U_x \) such that \(U_x \cap \{f = 0\} \cap \{T^n f = 0\} = \emptyset\) for some \(n > 0\), then \(\mu(U_x) = 0\).

In addition, either \((1)\) holds true if \(n > 0\), \(\mu(U_x) = 0\) if \(n = 0\). If in addition, either \((2)\) holds true, then \(\mu(U_x) = 0\) for all \(x \in \Omega\). If \(f = 0\) in a neighborhood of \(x \in \Omega\), then \(\mu(U_x) = 0\).

\[ \frac{d}{dt} \int_{\Omega} |f|^2 \, d\mu = \int_{\Omega} f \cdot \nabla f \, d\mu. \]

Remark 3.1. Theorem 3.2 with \(f \in C^1(\Omega, \mathbb{R})\) satisfying \(\nabla f \neq 0\) on \(\Omega\) and \(f(0) = 0\) on \(\Omega\) is still valid under the assumption that \(f(0) = 0\) on \(\Omega\) and \(f(0) = 0\) on \(\Omega\).

**Theorem 3.2.** Assume that \(\Omega\) is a bounded open set in \(\mathbb{R}^d\) with \(d \geq 2\). Let \(f \in C^1(\Omega, \mathbb{R})\) be a \(C^1\)-smooth function. Then, for every \(x \in \Omega\), there exists a neighborhood \(U_x \) such that \(U_x \cap \{f = 0\} \cap \{T^n f = 0\} = \emptyset\) for some \(n > 0\).

Thus, the set of points \(x \in \Omega\) such that \(f(x) = 0\) and \(T^n f(x) = 0\) for some \(n > 0\) is of measure zero.

In view of Theorems 3.2 and 3.2, we see that the spreading speed is defined as

\[ \frac{d}{dt} \int_{\Omega} |f|^2 \, d\mu = \int_{\Omega} f \cdot \nabla f \, d\mu. \]
\[ x(0) = (1 - I) d \quad \text{and} \quad 0 < t < 0 \]

Theorem 4.2

Let \( v_0 = (1 - I) d \) and let \( t \) be the positive solution to the following system

\[ x(t) = \frac{\partial (x(t)v_0 + (x - v_0))(x(t) - v_0)}{\partial t} \]

\[ \frac{\partial}{\partial t} x(t) = \frac{\partial (x(t)v_0 + (x - v_0))(x(t) - v_0)}{\partial t} \]

\[ \text{where} \quad x(t) = \frac{x(t)}{v_0} + (x - v_0)(x(t) - v_0) \]

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\[ \frac{\partial}{\partial t} x(t) = \frac{\partial (x(t)v_0 + (x - v_0))(x(t) - v_0)}{\partial t} \]
By the theory in Section 3, we can show that system (6)

\[ \frac{\partial u}{\partial t} + a \cdot \nabla u = \nabla \cdot (\nabla u) \]

has a unique multiplier \( \mathbf{\Omega} \).

\[ u(x, t) = \sum_{n=1}^{\infty} \frac{2}{\pi n} \sin \left( \frac{n \pi x}{L} \right) \cos \left( \frac{n \pi t}{T} \right) \]

The problem on the asymptotic speed of propagation of interaction

\[ u(x, t) = (t/t_0)^{1/2} \left( 1 - \sum_{n=1}^{\infty} \frac{2}{\pi n} \sin \left( \frac{n \pi x}{L} \right) \cos \left( \frac{n \pi t}{T} \right) \right) \]

with \( t_0 \) a fixed positive constant, \( \varepsilon \) is the speed of sound, and \( L \) is the wave length.

\[ \varepsilon \frac{\partial^2 u}{\partial x^2} - \varepsilon (t + \varepsilon) \frac{\partial u}{\partial x} = \varepsilon \frac{\partial u}{\partial t} \]

1.4 a. For monotone vector diffuse model with spatial spread

\[ \nabla^2 u = \frac{1}{\varepsilon} \frac{\partial u}{\partial t} \]


where \( \varepsilon \) is a parameter.

\[ \nabla^2 u = \frac{1}{\varepsilon} \frac{\partial u}{\partial t} \]

We assume that

\[ \frac{\partial u}{\partial t} + a \cdot \nabla u = \nabla \cdot (\nabla u) \]

and \( \delta \) is the diffusion coefficient.

\[ \frac{\partial u}{\partial t} + a \cdot \nabla u = \nabla \cdot (\nabla u) \]

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\[ \frac{\partial u}{\partial t} + a \cdot \nabla u = \nabla \cdot (\nabla u) \]

with \( t_0 \) a fixed positive constant, \( \varepsilon \) is the speed of sound, and \( L \) is the wave length.

\[ \varepsilon \frac{\partial^2 u}{\partial x^2} - \varepsilon (t + \varepsilon) \frac{\partial u}{\partial x} = \varepsilon \frac{\partial u}{\partial t} \]

It was proved in [601] (1976).
Theorem 4.1 (as defined in Theorem 4.9) of the paper is stated as follows:

If \( T \) is a solution of the equation \( (\cdot')_t + T = \int (\cdot') v \), then \((\cdot')_t = \int \frac{\partial \phi}{\partial \theta} \) for some \( \phi \) and \( \delta \), and \( \delta \) is a solution of the equation \( (\cdot')_v + \delta = \int \frac{\partial \phi}{\partial \theta} \).

Recall that a periodic traveling wave solution of the form \( (\cdot')_v = \int \frac{\partial \phi}{\partial \theta} \) exists if and only if \( \phi \) is a solution of the equation \( \phi' = \int \frac{\partial \phi}{\partial \theta} \).

For simplicity, we neglect the death of the population during the dispersal process and assume that the dispersal is constant. Assume further that the death rate is constant throughout the domain of the population. This simplification allows us to focus on the dynamics of the population within the chosen domain.

Consider a periodic traveling wave model with dispersal.

4.5. A monotone and periodic model with dispersal.

Theorem 4.5 (as defined in Theorem 4.9) of the paper is stated as follows:

If \( \delta \) is a solution of the equation \( (\cdot')_v + \delta = \int \frac{\partial \phi}{\partial \theta} \), then \( \delta \) is a solution of the equation \( \delta'' = \int \frac{\partial \phi}{\partial \theta} \).

Define

\( \delta(\cdot, \cdot) = \int \frac{\partial \phi}{\partial \theta} \) for some \( \phi \) and \( \delta \), and \( \delta \) is a solution of the equation \( \delta'' = \int \frac{\partial \phi}{\partial \theta} \).

Let \( \delta = \int \frac{\partial \phi}{\partial \theta} \) for some \( \phi \) and \( \delta \), and \( \delta \) is a solution of the equation \( \delta'' = \int \frac{\partial \phi}{\partial \theta} \).

Theorem 4.5 (as defined in Theorem 4.9) of the paper is stated as follows:

If \( \delta \) is a solution of the equation \( \delta'' = \int \frac{\partial \phi}{\partial \theta} \), then \( \delta \) is a solution of the equation \( \delta'' = \int \frac{\partial \phi}{\partial \theta} \).

Define

\( \delta(\cdot, \cdot) = \int \frac{\partial \phi}{\partial \theta} \) for some \( \phi \) and \( \delta \), and \( \delta \) is a solution of the equation \( \delta'' = \int \frac{\partial \phi}{\partial \theta} \).

Let \( \delta = \int \frac{\partial \phi}{\partial \theta} \) for some \( \phi \) and \( \delta \), and \( \delta \) is a solution of the equation \( \delta'' = \int \frac{\partial \phi}{\partial \theta} \).

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If \( \delta \) is a solution of the equation \( \delta'' = \int \frac{\partial \phi}{\partial \theta} \), then \( \delta \) is a solution of the equation \( \delta'' = \int \frac{\partial \phi}{\partial \theta} \).

Define

\( \delta(\cdot, \cdot) = \int \frac{\partial \phi}{\partial \theta} \) for some \( \phi \) and \( \delta \), and \( \delta \) is a solution of the equation \( \delta'' = \int \frac{\partial \phi}{\partial \theta} \).

Let \( \delta = \int \frac{\partial \phi}{\partial \theta} \) for some \( \phi \) and \( \delta \), and \( \delta \) is a solution of the equation \( \delta'' = \int \frac{\partial \phi}{\partial \theta} \).
The theory of spreading speeds and traveling waves for monotone perturbation of the advection reaction-diffusion model with bistable nonlinearity was established in [16] and applies to a non-local periodic reaction-diffusion equation. The bistable equation arises in population biology to model the invasion of an introduced species. The bistable equation is given by

\[ u_t = D u_{xx} + f(u), \quad x \in \mathbb{R}, \quad t > 0, \]

where \( u(x,t) \) represents the population density at location \( x \) and time \( t \), \( D \) is the diffusion coefficient, and \( f(u) \) is a bistable nonlinearity. The equation is posed on the real line \( \mathbb{R} \) and \( u \) satisfies zero-flux boundary conditions.

The bistable equation has two stable steady states: \( u = 0 \) and \( u = 1 \). The spreading speed is defined as the minimal wave speed \( c^* \) such that a traveling wave \( u(x,ct) \) with speed \( c \) connects these two states for all \( c \geq c^* \). For the bistable equation, this speed can be found by solving the eigenvalue problem associated with the linearized operator around the traveling wave. The spreading speed is the minimal wave speed at which the unstable manifold of the trivial solution connects to the first eigenfunction of the operator.
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