University of South Alabama **Fall Semester 2011**
Y. Sommerhäuser **MA 540: Sheet 4**

Differential Geometry

1. The curve
$$c(t) := (1 + \cos(t), \sin(t), 2\sin(t/2))$$
is called Viviani's curve.

   (a) Show that Viviani's curve is contained in the intersection of the sphere with radius 2 around the origin and the cylinder of radius 1 whose central axis is determined by the conditions $x = 1$ and $y = 0$.

   (b) Conversely, show that every point in this intersection is of the form $c(t)$ for a suitable t.

   Notes: A picture of Viviani's curve can be found on Wikipedia or in Exercise 20 to Section 13.1 on page 744 of the first edition of Rogawski's calculus book. (6 points)

2. (a) Show that the curvature of Viviani's curve is given by
$$\kappa(t) = \frac{\sqrt{13 + 3\cos(t)}}{\sqrt{3 + \cos(t)}^3}$$

   (b) Show that the torsion of Viviani's curve is given by
$$\tau(t) = \frac{6\cos(t/2)}{13 + 3\cos(t)}$$

   Notes: These computations are involved. (8 points)

3. Show by explicit, direct computation that for Viviani's curve we have
$$\|\dot{c}(t)\| \frac{\tau(t)}{\kappa(t)} = \frac{d}{dt}\left(\frac{\dot{\kappa}(t)}{\|\dot{c}(t)\|\tau(t)\kappa(t)^2}\right)$$

   Hints: Show first that
$$\frac{1}{\kappa(t)^2} + \left(\frac{1}{\|\dot{c}(t)\|\tau(t)}\frac{d}{dt}\left(\frac{1}{\kappa(t)}\right)\right)^2 = 4$$

   Then, without returning to trigonometric functions, obtain the assertion by differentiation. Also, note that $(1/\kappa)^{\cdot} = -\dot{\kappa}/\kappa^2$, and it is easier to work with the radius of the osculating circle $1/\kappa$ than with κ here. (6 points)

Due date: Wednesday, September 21, 2011. Please write your solution on letter-sized paper, and write your name on your solution. It is not necessary to submit this sheet with your solution.