University at Buffalo
Yorck Sommerhäuser

Spring Semester 2014
MTH 819: Sheet 9

Hopf Algebras

On this exercise sheet, the base field K is assumed to be algebraically closed of characteristic zero.

Problem 1: Suppose that G is a finite group and that $H = K[G]^*$ is the dual group ring considered in Problem 1 on Sheet 3.

1. Determine the irreducible characters.
2. Show that the character ring $\mathrm{Ch}(H)$ and the group ring $K[G]$ are isomorphic as algebras. (2 points)

Problem 2: Suppose that H is a semisimple Hopf algebra for which the character ring $\mathrm{Ch}(H)$ is a Hopf subalgebra of H^*. Show that there is a finite group G such that $H \cong K[G]^*$. (6 points)

Problem 3: For a semisimple Hopf algebra H, the character ring $\mathrm{Ch}(H)$ is a subalgebra of H^*. Decide whether H^* is free as a left $\mathrm{Ch}(H)$-module.
(Remark: We have seen in Problem 2 that the character ring is in general not a Hopf subalgebra of H^*, so the Nichols-Zoeller theorem does not apply.) (6 points)

Problem 4: Suppose that G is a finite group and that V is an irreducible representation of G. Deduce from the class equation for Hopf algebras that the dimension of V divides the order of G. (6 points)

Due date: Tuesday, April 8, 2014. Please write your solution on letter-sized paper, and write your name on your solution. It is not necessary to submit this sheet with your solution.