## Hopf Algebras

**Problem 1:** Show that the antipode of the Taft algebra $T$, considered in Problem 2 on Sheet 2, has order exactly $2n$.
(Remark: You need to show that the order is not smaller. This exercise shows that there are finite-dimensional Hopf algebras with antipodes of arbitrary even order.) (5 points)

**Problem 2:** Suppose that $H$ is a finite-dimensional Hopf algebra whose antipode has odd order. Show that this order is 1. (4 points)

**Problem 3:** If $H$ is a finite-dimensional Hopf algebra and $\Lambda \in H$ and $\lambda \in H^*$ are left integrals satisfying $\lambda(\Lambda) = 1$, show that the antipode can be computed from the integrals via the formula

$$S(h) = \Lambda_{(1)}\lambda(h\Lambda_{(2)})$$

(3 points)

**Problem 4:** Suppose that $H$ is a finite-dimensional commutative semisimple Hopf algebra over the algebraically closed field $K$. Show that there is a finite group $G$ so that $H \cong K[G]^*$. (8 points)

Due date: Tuesday, March 11, 2014. Please write your solution on letter-sized paper, and write your name on your solution. It is not necessary to submit this sheet with your solution.