University at Buffalo
Yorck Sommerhäuser

Spring Semester 2014
MTH 819: Sheet 13

Hopf Algebras

Problem 1: The symmetric group S_3 on three letters has a unique irreducible character of degree 2. Find the third Frobenius-Schur indicator $\nu_3(\chi)$ of this character. (4 points)

Problem 2: As we saw in Problem 2 on Sheet 11, the Kac-Palyutkin algebra, which was defined in Problem 1 on Sheet 10, also has a unique irreducible character of degree 2. Find the second Frobenius-Schur indicator $\nu_2(\chi)$ of this character. (6 points)

Problem 3: Suppose that H is a finite-dimensional semisimple Hopf algebra over a field of characteristic zero. The dual space $D(H)^*$ of its Drinfel'd double can be identified with $H \otimes H^*$ by considering $h \otimes \varphi \in H \otimes H^*$ as the linear form

$$(h \otimes \varphi)(\varphi' \otimes h') = \varphi'(h)\varphi(h')$$

If $\chi \in \mathrm{Ch}(D(H)) \subset D(H)^*$ is a character, we can therefore write it in the form

$$\chi = \sum_{i=1}^{r} h_i \otimes \varphi_i$$

Show that $\Phi(\chi) = \sum_{i=1}^{r} \varphi_i \otimes h_i$. (4 points)

Problem 4: Suppose that G is a finite group and that $H = K[G]$ is its group ring over the field K. For $g \in G$, let $C(g) \subset G$ be the centralizer of g, and let W be a $C(g)$-module.

1. Show that the induced module $V := H \otimes_{K[C(g)]} W$ becomes a module for the Drinfel'd double $D(H)$ if we define

$$(p_{g'} \otimes h').(h \otimes_{K[C(g)]} w) = \delta_{g', h'hgh^{-1}h'^{-1}} h'h \otimes_{K[C(g)]} w$$

2. Show that V is irreducible if W is irreducible. (6 points)

Due date: Tuesday, May 6, 2014. Please write your solution on letter-sized paper, and write your name on your solution. It is not necessary to submit this sheet with your solution.