University at Buffalo **Spring Semester 2014**
Yorck Sommerhäuser **MTH 819: Sheet 12**

Hopf Algebras

Problem 1: Suppose that H is a quasitriangular Hopf algebra and that u is its Drinfel'd element.

1. Show that $S^2(u) = u$.
2. Show that $g := u(S(u))^{-1}$ is group-like.
3. Show that $S^4(h) = ghg^{-1}$ for all $h \in H$. (6 points)

Problem 2: Suppose that H is a quasitriangular Hopf algebra with universal R-matrix R, that u is its Drinfel'd element and that $g := u(S(u))^{-1}$. Define $h := (\mathrm{id} \otimes \alpha^R)(R)$, where α^R is the right modular function.

1. Show that h is group-like.
2. Show that $g = ha^R$, where a^R is the right modular element. (6 points)

Problem 3: Suppose that H is a finite-dimensional Hopf algebra. The universal R-matrix of its Drinfel'd double $D(H)$ is

$$R = \sum_{i=1}^{n} (\varepsilon \otimes h_i) \otimes (h_i^* \otimes 1)$$

where $h_1, \ldots, h_n$ is a basis of H with dual basis $h_1^*, \ldots, h_n^*$. Show that

$$(\mathrm{id} \otimes \Delta)(R) = R_{13}R_{12}$$

(4 points)

Problem 4: Suppose that G is a finite group and that $H = K[G]$ is its group ring over the field K.

1. Show that the elements $p_g \otimes h$, for $g, h \in G$, form a basis of the Drinfel'd double $D(H)$, where we have used the notation from Problem 1 on Sheet 3.
2. Show that the product of these basis elements is given by

$$(p_g \otimes h)(p_{g'} \otimes h') = \delta_{g,hg'h^{-1}} p_g \otimes hh'$$

(4 points)

Due date: Tuesday, April 29, 2014. Please write your solution on letter-sized paper, and write your name on your solution. It is not necessary to submit this sheet with your solution.